# *Suggested Applications for Exploratory Factor Analysis to Conditions Encountered by Institutional Researchers*

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## Abstract

This paper provides empirically derived rationale for the application of Ordinary Least Squares factor analysis to data conditions that institutional researchers encounter as they analyze increasingly complex data sets. A Monte Carlo method was used to simulate data under 540 research conditions that varied in sample size, communality, dichotomization, and number of factors in the population. This paper also includes a general description of exploratory factor analysis in terms of theory and practice.

## Introduction

In the most basic sense, factor analysis is a set of procedures that researchers can employ to account for complex patterns of covariation among observed random variables. When used properly, these procedures yield a set of common factors and information concerning the extent to which factors "represent the original variables" (Henson & Roberts, 2006, p. 396). Through the use of factor analytic techniques, researchers derive meaningful and parsimonious explanations for large sets of observations (Tucker & MacCallum, 1997).

Although exploratory factor analytic strategies are useful in addressing both substantive and measurement objectives, these methods are open to criticism due to the "inherent subjectivity of the decisions" that researchers make when conducting their analyses (Henson & Roberts, 2006, p. 396). For example, without referring to criterion variables, researchers select matrices of association, factor extraction methods, criteria for retaining factors, factor rotation strategies, and coefficients for interpretation (Henson & Roberts, 2006). Attempts to incorporate non-normal data types into factor analytic research represent another potential area of criticism (Yuan, Marshall, & Bentler, 2002). While guiding principles are well established for analyzing continuous variables that exhibit multivariate normality, “no firm guidelines have as yet emerged concerning situations in which qualitative and quantitative variables are mixed together” (Krzanowski, 1983, p. 235). This lack of methodologically derived recommendations for subjecting dichotomously scored variables to factor analyses represents the problem to be addressed by this study.

This paper is intended to provide social scientists (in general) and institutional researchers (specifically) with empirically based guidance as they conduct factor analytic studies with data sets that contain mixtures of categorical and continuous variables. To enhance the potential utility of this study, the research focused on factor extraction methods commonly employed by social scientists. These methods include Principal Axis Factor analysis, Ordinary Least Squares factoring, and standard Maximum Likelihood (Conway & Huffcutt, 2003; Costello & Osborne, 2005; Fabrigar et al., 1999).

**Rationale**

The availability of “large administrative data sets” provides institutional researchers with opportunities to develop interpretable models of student behavior, evaluate existing predictive models, and (ultimately) influence policy decision making processes (Einav & Levin 2014, p. 6). Although these new data sets include “an almost unlimited set of individual-level behavioral characteristics,” these data will have many, ambiguously defined dimensions and extracting meaningful constructs or factors from their correlations will be challenging (Einav & Levin, 2014 p. 6). Currently, to make effective use of these data, researchers have to resolve research design questions with few, or no, empirically derived guidelines (Einav & Levin 2014; Krzanowski, 1983; Yuan, Marshall, & Bentler, 2002).

The evolution of large data sets, the variety of objectives to which researchers apply factor analytic techniques, and the lack of empirically derived guidelines for analyzing non-normal data sets provide this study with rationale. Each of these methodological conditions contribute to emerging opportunities for social scientists and institutional researchers. However, they also foster less than ideal practices as researchers attempt to develop useful models from increasingly complex relationships among their observations.

**Theoretical Framework**

While the work of Pearson represents the mathematical foundation for classical factor analysis, Spearman provided the initial description of statistical treatments associated with Principal Axis analyses (Harman, 1976). Spearman's work in identifying the strength and direction of relationships among intellectual abilities introduces a method for extracting common factors from correlations (Spearman, 1904). Formally, a common factor is "an internal attribute which affects more than one surface attribute in the selected set, or battery" (Tucker & MacCallum, 1997, pp. 2-3). A specific factor is systematic, but it influences only one variable in a given data set; measurement error is not a factor as it is neither internal nor systematic (Tucker & MacCallum, 1997).

The "traditional" or "classical" factor analysis model is defined by the following expression:

$$Z_j = a_{j1}F_1 + a_{j2}F_2 + \cdots + a_{jm}F_m + u_jY_j \quad (j = 1, 2, \cdots, n) \qquad (1)$$

In this model, an observed variable $Z_j$ is described by a linear combination of common factors

$(F_1, F_2, \cdots, F_m)$, and a unique factor, $u_j Y_j$. The $a$'s represent coefficients or loadings for the common factors; the number of common factors ($m$) is normally smaller than the number of observed variables, *n* (Harman, 1976). When considering the value of a specific variable, *j*, for a given individual, *i*, the factor model can be written as:

$$Z_{ji} = \sum_{p=1}^{m} a_{jp} f_{pi} + u_j Y_{ji} \qquad (2)$$

Where $f_{pi}$ is the common factor *p* for individual *i*; $a_{jp} F_{pi}$ represents the contribution of the factor on the linear composite. The residual error is given by $u_j Y_{ji}$ (Harman, 1976).

Through applying an exploratory factor analytic strategy to appropriate data, researchers can estimate the values of factor loadings for a given set of factors. The general steps in conducting an exploratory factor analysis include:

1. Selection of observed variables and observations on those variables.
2. Development of correlation matrices that are appropriate for the data and researchers' intentions.
3. Extraction of factors from correlations; this can be understood as identification of groups of variables that correlate with each other more strongly than other variables within a data set.
4. Application of rotation strategies to enhance parsimony.
5. Interpretation of factor pattern and structure coefficients and deriving meaning from the resulting theoretical constructs.

The decisions that researchers make when implementing these steps define the type of factor analysis being conducted and influence the quality of the results (Henson & Roberts, 2006). The primary foci of this paper are the decision associated with steps one and three of the above process.

**Research Questions**

This study explored levels of agreement between factor pattern matrices in a simulated population and matrices developed through Principal Axis, Ordinary Least Squares, and Maximum Likelihood factor analytic techniques. The assessments were conducted in a variety of data contexts, and the levels of agreement were evaluated through measures of sensitivity to factors simulated in the population, per element factor agreement, and factor loading bias (MacCallum et al., 1999). These measures, or outcome variables, were used to answer the following research questions:

1. Which of the studied factor analytic strategies resolved factor loading patterns that were in better agreement with the patterns imbedded in the population?
2. How do varying ratios of categorical to continuous variables influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
3. How do all of the simulated data characteristics interact to influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?

## Review of Literature

One measure of the prevalence of factor analytic research designs in contemporary literature can be found in a recent analysis of literature published in PsycInfo over a two-year period; this survey of the literature yielded more than 1700 articles that involved exploratory factor analytic research (Costello & Osborne, 2005). The variety of purposes to which these factor analyses are applied also highlights the importance of this statistical tool. As Conway and

Huffcutt (2003) noted, social scientists employ exploratory factor analysis to refine measurement tools, establish construct validity, and test hypotheses. The proliferation of exploratory factor analysis in social science research has served as justification for a number of studies that either attempt to establish research design guidelines or contrast typical practices with ideal reporting procedures (Costello & Osborne, 2005; Henson & Roberts, 2006; Krzanowski, 1983).

In their examination of 60 factor analytic studies, Henson and Roberts (2006) explored the quality of methodological decisions made by researchers and the manner in which they reported their results. This examination and similar evaluative studies highlight problematic practices in terms of the selection of matrices of association, rationale for retaining factors for interpretation, description of factor rotation methods, quantifying the amount of variance in distributions of observed variables accounted for by factors, and reporting of communalities among observed variables (Conway& Huffcutt, 2003; Henson & Roberts, 2006).

One methodological problem common in many studies includes selection of a principal components analysis (PCA) as a statistical treatment when the stated purpose of a study indicated that the researcher should have conducted an exploratory factor analysis (Conway & Huffcutt, 2003; Henson & Roberts 2006). Henson and Roberts (2006) noted that the majority of articles included in their study included objectives that warranted an exploratory factor analytic design; however, nearly 57% of the researchers engaged in principal components analysis. According to the Conway and Huffcutt (2003) study of published factor analytic studies, principal components analysis (PCA) was the most frequently cited method for analyzing data; however in 28% of the articles, researchers failed to describe either their intentions behind their analyses or methods for extracting factors (Conway & Huffcutt, 2003). As a suggested rationale for this problematic model selection, the authors noted that principal components analysis was the “default option for most statistical software packages” (Henson & Roberts, 2006, p. 403).

In addition to highlighting less than ideal design and reporting practices, a review of the methodological literature describes problems that researchers encounter as they incorporate dichotomously scored variables into their studies and the potential value of these complex data sets to social science research (Lin & Clayton, 2005; Mislevy, 1986; Osh & Lee, 2001; Sammel, Ryan & Legler, 1997). For example, by including dichotomous data sets into exploratory factor analyses, researchers reduce large contingency tables into more interpretable tables of fewer dimensions (Bartholomew, 1980); however, when “testing and estimating the reliability of a measured trait,” dichotomously measured variables provide researchers with difficulties in determining required sample sizes to achieve specified reliability coefficients (Donner & Eliasziew, 1994). Moreover, contexts that include “truly dichotomous outcome variables and continuous constructs that have been dichotomized have less power than contexts that contain continuous outcome variables exclusively (Donner & Eliasziew, 1994). Whether the data are dichotomous or continuous, most factor extraction methods require distributions of observed variables to exhibit multivariate normality, and this assumption cannot be maintained when studies rely on dichotomously scored observations (Krzanowski, 1982; Mislevy, 1986; Stevens, 2002; Yalcin & Amemiya, 2001).

A review of institutional research literature highlights several examples of researchers applying factor analytic techniques to their explorations of relationships among student demographic characteristics, student motivation, institutional policies, student engagement, and measures of desired educational outcomes (Allen, 1997; Bryan, Eagle, Wright, & Icenogle, 2012; Hossler, Ziskin, Moore, & Wakhungu 2008; Luo & Jamieson, 2005). For example, Sherman (1978) collected data from the Association of American Medical Colleges' Institution Profile System (IPS), Liaison Committee on Medical Education (LCME) questionnaire (all parts), the medical school information system (MSIS), and the Faculty Roster System (FRS) to explore the relationships among characteristics of medical school applicants and institutions. The resulting data set included measurements on 60 variables taken from 81 medical schools. As Sherman concluded, the number of observed variables would render any attempt to interpret all of the pairwise correlations so complicated as to be "prohibitive" (Sherman, 1978, p. 3). However, based on the assumption that a set of underlying, latent traits influenced measurements on these variables (and their resulting correlations), Sherman employed exploratory factor analysis to develop an interpretable 13 factor model of the relationship between student and institution (Sherman, 1978).

To examine the efficacy of statistical tools and the impact of data conditions on the quality of results from potential studies, researchers engage in Monte Carlo experiments (Robey, 1990). In this class of experiments, researchers employ computer programs to generate "vectors of random variates" that are analogous to samples of data (Robey, 1990, p. 278). Historically, methodologists employ these methods to explore type I error rates for different tests of significance when assumptions associated with those tests are not maintained (Robey & Barcikowski, 1992). The methodological literature illustrates the utility of these strategies as researchers examine a variety of design considerations inherent in factor analytic research.

Briggs and MacCallum (2003) compared Maximum Likelihood (ML) and Ordinary Least Squares (OLS) factor analysis methods in their capacities to recover "relatively weak common factors" (p. 26). To make this comparison, the researchers simulated two types of data sets that varied in term of the number of imbedded factors and manifest (observed) variables. Factors with loadings of .45 or lower were defined as weak. The three factor model contained one weak factor, and the four factor model contained two weak factors; the researchers embedded multiple levels of measurement and sample error in their simulations (Briggs & MacCallum, 2003). For each research condition, the researchers generated 1,000 correlation matrices.

To assess the accuracy of estimates derived by the tested factor analysis methods, the researchers employed a coefficient of congruence between the population and sample factor loadings for weak factors and a root mean square deviation calculated for weak factors only (Briggs & MacCallum, 2003). The results of their study indicated that, under a majority of conditions, OLS outperformed ML factor analysis in recovering weak factors. The advantage of OLS over ML became especially pronounced in the two-weak factor design; in these conditions, ML did not recover the fourth factor at all (Briggs & MacCallum, 2003).

Through an extensive review of literature and two novel simulation studies, Velicer and Fava (1998) examined the impact of subject and variable sampling on the quality of factor analysis solutions. The researchers' stated intent is "to determine the conditions that are likely to produce patterns that closely approximate the population patterns" (Verlicer & Fava, 1998, p. 233). In the first simulation study, the researchers generated population correlation matrices that contained varying levels of factor pattern complexity. Simplicity implied equal loadings, and increased complexity involved unequal loadings. The researchers also manipulated the variable to factor ratios and sample sizes. Because the authors understood that constraining the loadings to be equal on all salient variables was "highly artificial" (Velicer & Fava, 1998, p. 241), they conducted a second study in which the loadings varied in the population. The analytical methods included in these stimulation studies were principal components analysis, image component analysis, and Maximum Likelihood factor analysis (Velicer & Fava, 1998). The comparisons between sample and population factor loading matrices were evaluated through a root mean squared error (*g*). The mean values of *g* and the standard deviations of *g* were subjected to an analysis of variance (ANOVA).

In terms of the size of subject samples, the authors confirmed the results of early studies: “the most critical conditions for determining the degree of similarity between a sample pattern and the corresponding population pattern are the square root of the sample size and the average loading” (Velicer & Fava, 1998, p. 243). The results indicated that ICA “was clearly inferior to both MLFA and PCA” (Velicer & Fava, 1998, p. 245), and, when samples were either very large or very small, PCA was superior to MLFA.

Ogasawara (2000) explored methods for evaluating the variability in parameter estimates derived from factor analysis and component analysis. This exploration included “asymptotic correlations” between parameter estimates in factor analysis and principal component analysis, correlations for standardized variables, and “mean squared canonical correlation between factors and components. . .” (Ogasawara, 2000, p. 168); the author also included asymptotic standard errors for the estimates of the correlations between factors and principal components. Through a Monte Carlo simulation study and the use of existing data sets, the author tested the accuracy of these measures.

The simulation consisted of eight manifest variables and one factor; the sample size was 300; the simulation procedure was repeated 1000 times. The previously developed data set consisted of a correlation matrix of for six variables with a sample size of 220 (Ogasawara, 2000). The results from the simulation study and the study conducted on the existing data yielded large asymptotic correlations between estimates for factor analysis and the corresponding components analysis; the results yielded small standard errors associated with the canonical correlations. When the variances of the unique factors approach sphericity, “the two sets of results give similar interpretations for factors and components” (Ogasawara, 2000, p. 182).

In their investigation of the relationship between sample size and the accuracy of factor analytic solutions, Hogarty, Hines, Kromrey, Ferron, and Mumford (2005) conducted a Monte Carlo study to evaluate the minimum sample size requirements suggested in the methodological literature. The researchers compared factor pattern matrices extracted through Principal Axis Factor analysis from generated correlation matrices with known factor pattern loadings. The researchers manipulated levels of communality, number of variables, number of common factors, and sample size; the researchers also examined the interactions among all independent variables. The factor solutions were evaluated in terms of “factor loading sensitivity,” “pattern accuracy” (general pattern accuracy, total pattern accuracy, and per element pattern accuracy), congruence coefficients, “factor score estimate accuracy,” and Root Mean Squared Error (RMSE) (Hogarty, et al., 2005, p. 206).

The researchers found that, under nearly all research conditions, factor solution sensitivity was excellent. In terms of general pattern accuracy, the conditions associated with high communality yielded the best solutions. Although the researchers found low total pattern accuracy in nearly all research conditions, their results indicated that this accuracy increased as sample size and communality increased (Hogarty, et al., 2005). Their results indicated that per element accuracy was positively correlated with sample size, communality, and variable to factor ratio (Hogarty, et al., 2005). The results also highlighted a negative correlation between sample size and RMSE (Hogarty, et al., 2005). The researchers concluded that no minimum sample size or subject to variable ratio ensured “good factor recovery;” moreover, their results indicated that the quality of factor solutions was more strongly related with communality than sample size (Hogarty, et al., 2005, p. 223).

## Methods

A Monte Carlo method was used to simulate data under 540 different conditions; specifically, this study included a three (number of variables) by three (number of common factors) by three (communality levels) by four (sample sizes) by five (ratios of categorical to continuous variables) design. The strategies used to generate correlation matrices in this study were derived from Tucker, Koopman, and Linn's (1969) examination of factor analytic methods. The sampling methods were based on the strategies employed by Hogarty, et al. (2005).

Factor loading matrices derived through the tested factor extraction methods (sample) were compared to the known factor structure incorporated into the simulated population; sample factor loading matrices were evaluated through measures of factor loading sensitivity, agreement between sample and population factor patterns, and statistical bias (Hogarty et al., 2005; MacCallum, Widaman, Zhang, & Hong, 1999). The tested factor extraction models included Maximum Likelihood, Ordinary Least Squares, and Principal Axis Factor analyses.

### Factor Extraction Methods

**Principal Axis Factor analysis.** Principal Axis Factor analysis and principal components analysis are computationally similar (Stevens, 2002). When conducting a Principal Axis Factor analysis, researchers focus on a reduced correlation matrix as the matrix of association. This type of correlation matrix contains communality estimates on the main diagonal as opposed to ones (Cureton & D'Agostino, 1983; Harman, 1976; Stevens, 2002). A frequently used prior communality estimate of a variable, $z_t$, is its squared multiple correlation with all of the other variables in an instrument (Cureton & D'Agostino, 1983; Harman, 1976):

$$SMC_t = 1 - \frac{1}{r^{tt}} \tag{3}$$

Where $r^{tt}$ is the diagonal element of an inverse correlation matrix that corresponds to the variable $z_t$ (Harman, 1976).

The sum of squared factor coefficients gives the communality of a particular variable; as highlighted by the expression (1), $a_{j1}^2$ quantifies the first factor's contribution to the communality of variable $Z_j$ (Harman, 1976). The coefficients for the first factor are selected to maximize the factor's contribution to the total communality, $V_1$; this sum is given by (Harman, 1976):

$$V_1 \; = \; a_{11}^2 + a_{21}^2 + \cdots + a_{n1}^2 \tag{4}$$

Lagrange multipliers are used to develop a system of "characteristic equations" (Harman, 1976, p. 137). The roots of a characteristic equation are eigenvalues, and the largest root is the maximum value of the first factor's contribution to the total communality (Cureton & D'Agostino, 1983; Harman, 1976).

The next step in the Principal Axis Factor method includes determining the coefficients for the second factor. These coefficients are selected to maximize the factor's contribution to the remaining, or residual, communality. In a fashion similar to finding the coefficients for the first factor, the second largest root is equivalent to the root, or eigenvalue, of the first factor's residual communality matrix. This root of this residual matrix is equivalent to the second largest eigenvalue of the original, reduced correlation matrix (Cureton & D'Agostino, 1983; Harman, 1976). This procedure proceeds until the entire matrix of factor coefficients is developed.

**Ordinary Least Squares.** The primary goal of Ordinary Least Squares (OLS), also known as the Unweighted Least Squares or MINRES (Obenchain, 1975), method for obtaining factor solutions is to minimize the sum of squared differences between the observed and implied covariance matrices (Briggs & MacCallum, 2003). OLS solutions are derived through an iterative Principal Axis computational method that assigns weights to residuals of large and small factors equally (Briggs & MacCallum, 2003; Cureton & D'Agostino, 1983; Harman, 1976).

The Ordinary Least Squares procedure begins with a matrix of factor coefficients that, when multiplied by its transpose, yields a matrix of reproduced correlations with communalities on the principal diagonal (Cureton & D'Agostino, 1983; Harman, 1976). The next phase in the procedures involves fitting the reproduced correlation matrix to the matrix of observed correlations. This is achieved through minimizing an objective function that is based on the sum of squares of the off-diagonal residuals between the observed and reproduced correlation matrices (Harman, 1976). Through an iterative process of adding "displacements" to each element of the original factor loading matrix, a new factor matrix is developed (Harman, 1976, p. 177). The values of these displacements are selected so that the resulting matrix minimizes the objective function.

**Maximum Likelihood.** Based on the assumption that a specified number of factors exists in a population, Maximum Likelihood factor analysis yields estimates of factor loadings for a given sample size and number of observed variables (Harman, 1976). The basic factor model defined in the theoretical framework can be restated as

$$x = \mu + \Lambda f + \varepsilon \qquad (5)$$

As defined above, $x$ is a column of observed variables, $\mu$ is the mean vector of observed variables, $f$ is a column vector of common factors, $\Lambda$ is a matrix of factor loadings, and $\varepsilon$ is a vector of unique factors (Chen, 2003). The population covariance matrix, $\Sigma$, is given by the following expression:

$$\Sigma = \Lambda\Lambda' + \Psi \qquad (6)$$

Where Ψ is a diagonal matrix of unique variances (Chen, 2003). Because the observed covariance matrix, S, can be calculated from the sample and is "an unbiased estimate of Σ," the factor loadings and unique variances are all that must be estimated from the sample (Chen, 2003, p. 310). The parameters are estimated by maximizing a likelihood function or minimizing a corresponding function (Chen, 2003; Cureton & D'Agostino, 1983; Harman, 1976). These expressions are functions of the elements of Λ and Ψ matrices (Chen, 2003; Harman, 1976). If the observed variables exhibit multivariate normality, Maximum Likelihood strategies can yield estimators that utilize all the information from the sample and contain small limiting variances (Chen, 2003; Harman, 1976). Moreover, as sample sizes increase, "the estimators will converge (in a probabilistic sense) to the true parameters" (Harman, 1976, p. 198).

Maximum Likelihood strategies are scale-invariant; both covariance matrices and their corresponding correlation matrices will yield the same factor patterns (Cureton & D'Agostino, 1983). Maximum Likelihood techniques provide researchers with both parameter estimates and statistical indicators of model adequacy (Conway & Huffcutt, 2003; Harman, 1976; Mislevy, 1986). However, these strategies are dependent on the assumptions that, in addition to the observed variables, the common factors exhibit multivariate normality.

**Simulation Method**

The simulation process included a "mathematical, probabilistic model" and presumed the existence of major, minor, and unique factors. The major factors represent the "influences on observed scores of individuals for the phenomena which the experimenter wishes to study" (Tucker, Koopman, & Linn, 1969, p. 424); minor factors exert systematic influence on the value of observations but are not within the experimenters' control, and unique factors represent error. Major factors are identified by a subscript value of one; minor factors are given a subscript value of two, and a subscript of three indicates a unique factor. The number of each type of factor is designated by $M_s$ (Tucker, Koopman, & Linn, 1969).

The generation of correlation matrices begins with a matrix $A_s$ of “actual input factor loadings” (Tucker, Koopman, & Linn, 1969, p. 425). Through a three-step process, these actual input loadings were derived from a matrix of conceptual input loadings, $\tilde{A}$. Conceptual input factor loadings represent the researcher’s expectations concerning the “factorial composition of the variables” (Tucker, Koopman, & Linn, 1969, p. 426).

The first step in the development of conceptual input loadings involved the creation of “relative conceptual input loadings” for each variable. For a three-factor domain, the loadings conform to the following guidelines:

1. A zero, one, or two is chosen at random and is assigned to the first factor.
2. The sum of the loadings for any one variable is limited to two; this limit implies that if the first loading is two, then other two must be zero; if the first loading is one, then the other two have an equal probability of being a zero or a one.
3. The loading of the third factor is chosen so that the sum of all three will be two (Tucker, Koopman, & Linn, 1969).

Translating conceptual input factor loading matrices into matrices of actual input factor loadings is accomplished through a three-step process. In the first step, the conceptual input factor loadings are combined with “random normal deviates;” these deviates represent the natural “discrepancies” that occur in the construction of instruments (Tucker, Koopman, & Linn, 1969, p. 428). The output of this step, $(y_1)_{im_1}$, is defined by:

$$(y_1)_{jm_1} = (\tilde{a}_1)_{jm_1} c_{m1} + d_{1j} x_{1m_1} (1 - c_{m1}^2)^{1/2} \quad (7)$$

Where:

1. $(\tilde{a}_1)_{jm_1}$is the entry in row j and column $m_1$ of matrix $\tilde{A}_1$
2. $x_{jm_1}$ is a random, normal deviate ($\mu = 0,\ \sigma = 1$)
3. $c_{m_1}$ is a constant for each factor $m_1$; the possible values range from zero to one; the constants represent the "general control an experimenter has on the loading of actual variables on the factors" (Tucker, Koopman, & Linn, 1969, p. 429)
4. $d_{1j}$ is a constant for each variable j; this constant normalizes each row of $x_{1m_1}$to a unit length vector; it is defined as: $d_{1j} = \left(\sum_{m_1} x_{jm_1}^2\right)^{-1/2}$(Tucker, Koopman, & Linn, 1969, p. 429)

The second step in this translation process includes a skewing function that reduces negativity in factor loadings. This function yields coefficients, $(z_1)_{jm_1}$, according to the following equality:

$$(z_1)_{jm_1} = \frac{(1+k)}{(2+k)} \frac{(y_1)_{jm_1}\left[(y_1)_{jm_1} + \left|(y_1)_{jm_1}\right| + k\right]}{\left[\left|(y_1)_{jm_1}\right| + k\right]} \tag{8}$$

Where k is a parameter that can range from zero to infinity. Each vector of $(z_1)_{jm_1}$coefficients is reduced to unit length by the following:

$$(a_1^*)_{jm_1} = g_{1j}(z_1)_{im_1} \tag{9}$$

where $g_{1j} = \left[\sum_{m_1}(z_1)_{jm_1}^2\right]^{-1/2}$

The third step in this process includes scaling the matrix "to ensure desired levels of communality" (Hogarty et al., 2005, p. 207).

The matrix of actual input factor loadings, $A_s$ is a $J \times M_s$ matrix that contains a row for each variable J and a column for each major, minor, and unique factor (Tucker, Koopman, & Linn, 1969, p. 425). For each matrix $A_s$, a matrix $A_s^*$ can be defined by adjusting the rows of $A_s$

to unit length vectors. P is a square, symmetric matrix of order J; P is positive and semi-definite; it is defined by:

$$P_s = A_s^* A_s^{*\prime} \tag{10}$$

$$Diag(P_s) = I. \tag{11}$$

The simulated correlation matrix is given by:

$$R = B_1 P_1 B_1 + B_2 P_2 B_2 + B_3 P_3 B_3 \tag{12}$$

$B_s$ are diagonal matrices that include $b_{1i}$, $b_{2i}$, and $b_{3i}$as entries. These entries are real, positive numbers that have the following property:

$$b_{1i}^2 + b_{2i}^2 + b_{3i}^2 = 1 \tag{13}$$

These considerations imply the following equalities:

$$r_{ii} = 1 \tag{14}$$

$$Diag\ (R) = I \tag{15}$$

Matrix $A_s$is now defined as:

$$A_s = B_s A_s^* \tag{16}$$

The correlation matrix is given by:

$$R = A_1 A_1' + A_2 A_2' + A_3 A_3' = (A_1, A_2, A_3)(A_1, A_2, A_3)' \tag{17}$$

The supermatrix ($A_1$, $A_2$, $A_3$) contains the matrices $A_1$, $A_2$, and $A_3$ as horizontal sections (Tucker, Koopman, & Linn. 1969).

The coefficients in the $B_s$ matrices “regulate” the amount of variability in the variables that is related to the major, minor, and unique factors. The $B_1^2$ matrix contains communalities, and the $B_3^2$contain values for uniqueness. When $B_2$ matrix is zero, the “simulation model” equals the “formal model” (Tucker, Koopman, & Linn, 1969, p. 426); this study included the formal model as the simulation model.

Because the $B_2$ matrix was set to zero, the input factor loadings for minor factors were zero. This forced the “data generation model” to match “a factor analytic model” with the number of common factors equal to the levels specified for each combination of research contexts that was examined in this study. This study included two, four, and eight common factors.

This simulation was developed in SAS 9.3. To develop population correlation matrices and generate sample date, this study included a simulation strategy that is based on Hogarty, Hines, Kromrey, Ferron, & Mumford’s (2005) investigation of the relationship between sample size and factor solutions. The samples developed through the simulated correlation matrices included distributions that contained varying levels of categorical variables; these levels included 5%, 25%, 50%, 75%, and 95% of the observed variables. While simulating a broad range of data contexts, these percentages resulted in whole numbers of variables in data sets that contain 20, 40, and 60 observed variables. When data were intended to represent categorical variables, the resulting, simulated value was dichotomized at 0.5. When the value was less than 0.5, the value of the categorical variable was set to zero; when the value was greater than or equal to 0.5, the value of the variable was set to one.

This study incorporated participant (*N*) to observed variable (*p*) ratios that ranged from “insufficient to those considered more than acceptable” (MacCallum, Widaman, Zhang, & Hong, 1999, p. 92). The four sample size (*N*) conditions included 100, 200, 300, and 1000 simulated subjects. The resulting *N: p* ratios ranged from 1.67:1 to 30:1.

The simulated research contexts included two, four, and eight common factors. Because factors defined by fewer than two variables would “contradict the basic idea of a factor as a latent construct” (Henson & Roberts, p. 408), the upper limit on the number of common factors simulated in this study was constrained by the simulated context which includes 20 observed variables. In this study, the ratio of observed variables to common factors ranged from 2.5:1 to 30:1.

The simulation included three community levels: high, wide, and low communality ($h^2$) (Hogarty et al., 2005; MacCallum, Widaman, Zhang, & Hong, 1999). These communality levels correspond to the following:

1. High--$h^2$ for each variable are randomly drawn from values of .6, .7, and .8.
2. Wide--$h^2$ for each variable are randomly drawn from values of .2, .3, .4, .5, .6, .7, and .8.
3. Low--$h^2$ for each variable are randomly drawn from values of .2, .3, and .4.

The initial factor solutions from each model in each condition were subjected to a varimax rotation in all simulated contexts.

**Outcome Variables**

Loading sensitivity is expressed in terms of the proportion of variables that have factor pattern coefficients that are greater than or equal to .30 on at least one factor; this indicator measures the level of agreement between sample and population matrices in terms of variables that load on at least one factor (Hogarty et al., 2005).

The measure for per element pattern agreement is based on a P × K matrix. Elements of each matrix contained ones, indicating agreement, where the corresponding factor loading had an absolute value of .30 or greater in both the sample and population factor pattern matrices; elements of the matrix also contained ones when the variable had loadings of less than the absolute value of .30 on the corresponding factor in both the sample and population. When these criteria are not met, the element had a value of zero. The resulting measure of per element pattern agreement is the proportion of samples in which the agreement criteria were met for each observed variable by factor combination.

Statistical bias is included as an indicator of congruence between sample and population factor loadings is statistical bias. The P × K matrix is populated by estimates of statistical bias for each factor loading. These bias estimates were averaged across all samples.

**Analysis of Results**

The results of this study were examined through a repeated measures analysis of variance with a five-between and one-within group design (Stevens, 2007). The between groups factors are the manipulated variables that comprise the set of simulated research contexts. The within groups portion of the analysis includes the three factor extraction methods under consideration in this study. One repeated measures analysis of variance will be conducted for each of the three dependent variables included in this study; dependent variables include measures of factor loading sensitivity, factor pattern agreement, and factor loading bias.

To identify "practical differences" when statistical differences are found, effect sizes were examined (Stevens, 2007, p. 127). Specifically, this study employed a generalized eta-squared; this effect size parameter is defined by (Bakeman, 2005, p. 380)

$$\eta_G^2 = \frac{\sigma_{effect}^2}{\delta \times \sigma_{effect}^2 + \sigma_{measured}^2} \tag{19}$$

where:

1. $\sigma_{effect}^2$ is variance due to "manipulated factors," or between group variance (Bakeman, 2005, p. 380);
2. $\delta$ is equal to one if the effect includes only factors that are manipulated by the investigator;

3. $\sigma^2_{measured}$ includes variance due to individual differences (Bakeman, 2005);

In addition to the summary tables associated with balanced, between groups, factorial analyses of variance, results of this study are presented graphically. When statistically significant differences among factor extraction methods were identified, the Tukey procedure was employed to identify the sources of these significant differences.

## Results

### Factor Loading Sensitivity

Table 16 presents descriptive statistics concerning the univariate distribution of the loading sensitivity measure by each of the tested factor extraction methods. Observed levels of skewness, Kurtosis, and the Shapiro-Wilk tests of normality provide evidence of significant non-normality in the univariate distributions of factor loading sensitivity for all three factor extraction methods. Moreover, Mauchly's test of transformed variables, $\chi^2$ (2) = 337.428, $p < .0001$, indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To account for these violations of the assumptions associated with the univariate, repeated measures analysis of variance, the Greenhouse-Geisser adjustment to degrees of freedom was used to develop *p* values for the within subjects tests of significance.

Table 16
*Descriptive Statistics for Distribution of Factor Loading Sensitivity Values*

| Factor Extraction Method | *N* | *M* | *SD* | Skewness | Kurtosis |
|---|---|---|---|---|---|
| Principal Axis | 540 | 0.414 | 0.326 | 0.305 | -1.130 |
| Ordinary Least Squares | 540 | 0.993 | 0.008 | -1.122 | 0.158 |
| Maximum Likelihood | 540 | 0.391 | 0.300 | 0.254 | -1.120 |

The multivariate analysis of variance indicate that factor loading sensitivity differed significantly by factor extraction method main effect, Λ = .0001, $F$ (2, 495) = 342901, $p < .0001$. In addition, all interactions among factor extraction method and manipulated research characteristics yielded significant differences in factor loading sensitivity. As table A1 highlights (see Appendix A), eight of the ten first order interactions among main effects are also significant at the alpha = 0.025 level. Results of the repeated analysis of variance yield a similar pattern of significance as those associated with the multivariate analysis (see table A2 in Appendix A). The model including main effects and first-order interactions accounted for 99.9% of the variability associated with factor loading sensitivity.

The between subjects analysis yielded evidence that the factor loading sensitivity differed significantly for all of the main effects and five of the first-order interactions among main effects; these interactions include

1. Number of factors by number of observed variables (K × P), $F$ (8, 920) = 1086.34, $p < .0001$, $\eta_G^2 = .862$.
2. Number of factors by communality range (K × H), $F$ (4, 460) = 11.81, $p < .0001$, $\eta_G^2 = .062$.
3. Number of factors by level of dichotomization (K × D), $F$ (8, 460) = 5.43, $p < .0001$, $\eta_G^2 = .574$.
4. Number of observed variables by communality range (P × H), $F$ (8, 920) = 20.47, $p < .0001$, $\eta_G^2 = .059$.

Because at least one significant first order interaction includes each of the main effects, this study focuses on the interaction effects only (Cohen, 1988).

As table 17 demonstrates, the Ordinary Least Squares factor extraction method yields the highest mean proportions of variables that load on at least one factor in both the population and the sample at all levels of the interaction between number of factors and number of observed variables.  For the Principal Axis and Maximum Likelihood strategies, factor loading sensitivity is positively related to the number of factors and negatively related to the number of observed variables.

Table 17
*Means and Standard Deviations of Factor Loading Sensitivity by Factor Extraction Method and Number of Factors by Observed Variables Interaction (K x P)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x P)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 2 × 20 | .478 | 0.030 | .995 | 0.007 | .475 | 0.031 |
| 2 × 40 | .012 | 0.012 | .995 | 0.006 | .140 | 0.013 |
| 2 × 60 | .000 | 0.000 | .995 | 0.006 | .000 | 0.00 |
| 4 × 20 | .794 | 0.025 | .991 | 0.009 | .753 | 0.036 |
| 4 × 40 | .317 | 0.016 | .992 | 0.008 | .316 | 0.016 |
| 4 × 60 | .138 | 0.018 | .992 | 0.008 | .141 | 0.015 |
| 8 × 20 | .980 | 0.004 | .993 | 0.009 | .900 | 0.016 |
| 8 × 40 | .659 | 0.027 | .993 | 0.008 | .583 | 0.037 |
| 8 × 60 | .345 | 0.034 | .993 | 0.007 | .334 | 0.037 |

*N  = 540*

Table 18 provides means and standard deviations of factor loading sensitivity values based on the number of factors by communality range interaction. The Ordinary Least Squares factor extraction method yields the highest mean proportions of variables that load on at least one factor in both the population and the sample. For the Principal Axis and Maximum Likelihood strategies, the measure of factor loading sensitivity is positively related to both the number of factors and community range. Mean values of factor loading sensitivity associated with the Ordinary Least Squares factor extraction method appeared to be independent of the interaction effect values.

Table 18
*Means and Standard Deviations of Factor Loading Sensitivity by Factor Extraction Method and Number of Factors by Communality Range Interaction (K x H)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x H)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 2 × Low | .155 | 0.214 | .999 | 0.007 | .154 | 0.212 |
| 2 × Wide | .165 | 0.217 | .996 | 0.003 | .165 | 0.214 |
| 2 × High | .170 | 0.243 | 1.000 | 0.000 | .170 | 0.242 |
| 4 × Low | .409 | 0.289 | .982 | 0.006 | .390 | 0.259 |
| 4 × Wide | .412 | 0.266 | .993 | 0.002 | .398 | 0.246 |
| 4 × High | .429 | 0.284 | .999 | 0.000 | .422 | 0.276 |
| 8 × Low | .651 | 0.273 | .982 | 0.005 | .584 | 0.242 |
| 8 × Wide | .655 | 0.264 | .996 | 0.002 | .600 | 0.233 |
| 8 × High | .678 | 0.250 | .999 | 0.000 | .635 | 0.231 |

*N = 540*

Box and whisker plots indicate that factor loading sensitivity values associated with Ordinary Least Squares have smaller ranges and semi-interquartile ranges than the distributions associated with Maximum Likelihood and Principal Axis Factor extraction methods (see Figure 2 in Appendix B). For the Ordinary Least Squares factor extraction method, measures of both central tendency and dispersion in the distributions of factor loading sensitivity values appear to be independent of levels of the number of factors by communality interaction.

Table 19 presents means and standard deviations of factor loading sensitivity values based on the number of factors by level of dichotomization interaction. As was the case in the previous two interactions, factor loading sensitivity is highest for the Ordinary Least Squares factor extraction method across all levels of the interaction. For the Principal Axis and Maximum Likelihood strategies, the box plot highlights the positive relationship between factor loading sensitivity values and the number of factors (see Figure 3 in Appendix B).

For each of the number of factors levels, the mean values for factor loading sensitivity increase in value between the .05 and .75 levels of dichotomization. However, for each combination of effects, the mean factor loading sensitivity value decreases between the .75 and .95 dichotomization levels.

Table 19
*Means and Standard Deviations of Factor Loading Sensitivity by Factor Extraction Method and Number of Factors by Level of Dichotomization Interaction (K x D)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x D)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 2 × .05 | .155 | 0.213 | .994 | 0.007 | .155 | 0.211 |
| 2 × .25 | .160 | 0.223 | .995 | 0.006 | .160 | 0.222 |
| 2 × .50 | .165 | 0.228 | .995 | 0.006 | .165 | 0.226 |
| 2 × .75 | .170 | 0.237 | .995 | 0.006 | .170 | 0.235 |
| 2 × .95 | .167 | 0.230 | .996 | 0.006 | .167 | 0.229 |
| 4 × .05 | .413 | 0.288 | .992 | 0.008 | .398 | 0.266 |
| 4 × .25 | .418 | 0.285 | .991 | 0.009 | .405 | 0.267 |
| 4 × .50 | .423 | 0.287 | .992 | 0.007 | .412 | 0.271 |
| 4 × .75 | .414 | 0.274 | .992 | 0.008 | .402 | 0.254 |
| 4 × .95 | .413 | 0.274 | .991 | 0.009 | .399 | 0.253 |
| 8 × .05 | .646 | 0.271 | .993 | 0.008 | .584 | 0.247 |
| 8 × .25 | .656 | 00267 | .993 | 0.007 | .597 | 0.240 |
| 8 × .50 | .666 | 0.263 | .992 | 0.008 | .613 | 0.238 |
| 8 × .75 | .673 | 0.260 | .992 | 0.009 | .621 | 0.230 |
| 8 × .95 | .667 | 0.259 | .993 | 0.008 | .616 | 0.231 |

*N = 540*

Mean values for factor loading sensitivity associated with the interaction among numbers of observed variables and communality (P × H) are presented in Table 20. As was the case for all significant interactions, Ordinary Least Squares yields the highest mean proportion of variables that load on at least one factor in both the sample and the population. For Principal Axis and Maximum Likelihood factor extraction strategies, the mean values for the factor loading sensitivity measure are negatively related to the number of observed variables.

Table 20

*Means and Standard Deviations of Factor Loading Sensitivity by Factor Extraction Method and Number of Observed Variables by Communality Range Interaction (P x H)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(P x H)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 20 × Low | .745 | 0.220 | .983 | 0.008 | .693 | 0.184 |
| 20 × Wide | .739 | 0.211 | .995 | 0.002 | .696 | 0.179 |
| 20 × High | .767 | 0.198 | .999 | 0.001 | .741 | 0.175 |
| 40 × Low | .322 | 0.266 | .985 | 0.007 | .292 | 0.226 |
| 40 × Wide | .329 | 0.256 | .995 | 0.003 | .307 | 0.225 |
| 40 × High | .336 | 0.78 | .999 | 0.000 | .315 | 0.255 |
| 60 × Low | .147 | 0.133 | .986 | 0.006 | .144 | 0.126 |
| 60 × Wide | .162 | 0.140 | .995 | 0.003 | .160 | 0.138 |
| 60 × High | .173 | 0.158 | 1.000 | 0.000 | .170 | 0.155 |

*n = 540*

The box and whisker plots (see Figure 4 in Appendix B) demonstrate that factor loading sensitivity values associated with Ordinary Least Squares yielded smaller ranges and semi-interquartile ranges in factor loading sensitivity values than the distributions associated with

Maximum Likelihood and Principal Axis Factor extraction methods (see Figure 4 in Appendix B). For Ordinary Least Squares samples, mean values of factor loading sensitivity are negatively related to the number of observed variables but independent of communality level.

**Per Element Pattern Agreement**

Table 26 presents descriptive statistics concerning the distribution of the per element pattern agreement measure by each of the tested factor extraction methods. Shapiro-Wilks' tests of normality for per element pattern agreement for all three factor extraction methods yield evidence that the distributions do not conform to assumptions regarding univariate normality. Mauchly's test of transformed variables, $\chi^2(2) = 494.79$, $p < .0001$, indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To address the potential for increased Type I error rate, tests of significance associated with the repeated measures analysis of were be based on the Greenhouse-Geisser adjustment to degrees of freedom (Stevens, 2002).

Table 26
*Descriptive Statistics for Distribution of Per Element Pattern Agreement Values*

| Factor Extraction Method | N | M | SD | Skewness | Kurtosis |
|---|---|---|---|---|---|
| Principal Axis | 540 | .593 | 0.180 | -0.842 | -0.465 |
| Ordinary Least Squares | 540 | .750 | 0.750 | 0.005 | -0.819 |
| Maximum Likelihood | 540 | .605 | 0.185 | -0.892 | -0.445 |

The results of these multivariate analyses of variance indicate that per element agreement differed significantly by factor extraction method, $\Lambda = .008$, $F(2, 459) = 26828.4$, $p < .0001$. In addition to the method main effect, per element agreement values differed significantly by manipulated research charateristics. This analysis also indicated that per element agreement differed significantly across eight of the first-order interactions. The results of this analysis are presented in Table A3 (see Appendix A).

The univariate, repeated measures analysis of variance indicated that per element pattern agreement differed significantly based on factor extraction method, $F(2, 920) = 45801.9$, $p< .0001$, $\eta_G^2 = .934$. Results of the between subject portion of the analysis indicated that per element agreement differed significantly across all manipulated research characteristics (main effects) and all factor extraction methods. The model that included main effects and first-order interactions among main effects accounted for 98.7% of the variability in the observed data (see Table A4 in appendix A).

According to the results of within subjects analyses, the number of factors by observed variables, $F$ (8, 920) = 724.05, $p < .0001$, $\eta_G^2 = .474$, and the samples size by communality range, $F$ (12, 920) = 44.65, $p < .0001$ , $\eta_G^2 = .077$, were both statistically significant and yielded effect sizes that were medium or greater (Cohen, 1988). The between subject analysis indicated that the following interactions also met the criteria for continued analyses:

1. Number of factors by sample size, $F$ (6, 460) = 18.540, $p< .0001$, $\eta_G^2 = .172$
2. Number of factors by communality range, $F$ (4, 460) = 96.70, $p < .0001$, $\eta_G^2 = .419$
3. Number of observed variables by communality range, $F$ (4, 460) = 25.54, $p < .0001$, $\eta_G^2 =$ .155.

As Table 27 highlights, in all levels of the interaction between number of factors and observed variables, the Ordinary Least Squares factor extraction strategy yielded the highest mean levels of per element pattern agreement. Across all factor extraction methods, the box and whisker plots in Figure 5 (see appendix B) highlight the positive relationship between per element agreement and number of factors imbedded in the population; moreover, the differences among the three factor extraction methods, in terms of per element agreement, decreases as the number of factors increases.

Table 27
*Means and Standard Deviations of Per Element Pattern Agreement for Factor Extraction Methods by the Number of Factors and Number of Observed Variables Interaction (K x P)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x P)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 2 × 20 | .521 | 0.052 | .704 | 0.053 | .539 | 0.057 |
| 2 × 40 | .302 | 0.075 | .666 | 0.040 | .303 | 0.076 |
| 2 × 60 | .296 | 0.071 | .659 | 0.041 | .296 | 0.071 |
| 4 × 20 | .660 | 0.035 | .718 | 0.057 | .686 | 0.034 |
| 4 × 40 | .628 | 0.046 | .771 | 0.040 | .673 | 0.049 |
| 4 × 60 | .646 | 0.057 | .789 | 0.043 | .652 | 0.060 |
| 8 × 20 | .724 | 0.021 | .766 | 0.032 | .719 | 0.025 |
| 8 × 40 | .776 | 0.026 | .821 | 0.050 | .775 | 0.028 |
| 8 × 60 | .796 | 0.025 | .855 | 0.044 | .801 | 0.026 |

*N = 540*

As Table 28 and Figure 6 (See appendix B) illustrate, mean values of the per element pattern are positively related to both the number of factors and the sample size levels; differences in this measure among the three tested factor extraction strategies diminishes as the number of factors increases. The box and whisker plots in Figure 6 also highlight a convergence in mean and median values of per element factor agreement at the interaction level that includes eight factors and a sample size of 100. For Principal Axis and Maximum Likelihood methods, variability in the distributions of per element agreement are the greatest in the interactions that include two factors. Ordinary Least Squares yields the highest mean values of the per element pattern agreement across all levels of the number of factors by sample size (K × N) interaction.

Table 28

*Means and Standard Deviations of Per Element Agreement by Factor Extraction Method and Number of Factors by Sample Size Interaction (K x N)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x N)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 2 × 100 | .375 | 0.112 | .654 | 0.037 | .388 | 0.128 |
| 2 × 200 | .373 | 0.123 | .675 | 0.044 | .380 | 0.132 |
| 2 × 300 | .372 | 0.128 | .683 | 0.048 | .377 | 0.134 |
| 2 × 1000 | .371 | 0.137 | .694 | 0.057 | .372 | 0.138 |
| 4 × 100 | .631 | 0.038 | .712 | 0.052 | .654 | 0.042 |
| 4 × 200 | .652 | 0.042 | .756 | 0.046 | .669 | 0.048 |
| 4 × 300 | .660 | 0.045 | .772 | 0.043 | .675 | 0.051 |
| 4 × 1000 | .675 | 0.052 | .798 | 0.045 | .683 | 0.056 |
| 8 × 100 | .731 | 0.041 | .760 | 0.043 | .743 | 0.039 |
| 8 × 200 | .750 | 0.047 | .808 | 0.044 | .763 | 0.042 |
| 8 × 300 | .758 | 0.048 | .829 | 0.043 | .770 | 0.043 |
| 8 × 1000 | .770 | 0.047 | .860 | 0.042 | .782 | 0.041 |

*N = 540*

Table 29 provides means and standard deviations for these values of per element agreement for the number factors by level of communality (K × H) interaction. As was the case in the previous interactions, the mean per element pattern agreement values are highest for Ordinary Least Squares in all levels of the interaction. In general, the mean values for per element pattern agreement are positively related to the number of factors and negatively related to the communality range. However, as Figure 7 (see appendix B) highlights, the relationship between per element agreement and communality becomes positive in the eight factor condition for the Ordinary Least Squares method.

Table 29
*Means and Standard Deviations of Per Element Pattern Agreement for Factor Extraction Method and Number of Factors by Communality Range Interaction (K x H)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x H)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 2 × Low | .444 | 0.092 | .690 | 0.053 | .456 | 0.107 |
| 2 × Wide | .379 | 0.108 | .690 | 0.047 | .382 | 0.111 |
| 2 × High | .296 | 0.125 | .648 | 0.033 | .300 | 0.130 |
| 4 × Low | .697 | 0.032 | .775 | 0.075 | .721 | 0.021 |
| 4 × Wide | .654 | 0.022 | .758 | 0.047 | .667 | 0.021 |
| 4 × High | .612 | 0.040 | .745 | 0.033 | .622 | 0.042 |
| 8 × Low | .775 | 0.047 | .797 | 0.065 | .792 | 0.038 |
| 8 × Wide | .750 | 0.043 | .815 | 0.054 | .761 | 0.038 |
| 8 × High | .732 | 0.044 | .830 | 0.043 | .741 | 0.038 |

*N = 540*

Table 30 presents means and standard deviations for per element pattern agreement associated with the number of observed variables by level of communality (P × H) interaction. In all levels of this interaction, Ordinary Least Squares factor extraction method has the highest mean value of per element agreement. As evidenced by the box and whisker plots in Figure 8 (see Appendix B), the patterns of central tendency and dispersion in distributions of per element agreement for Maximum Likelihood and Principal Axis Factor extraction methods are nearly identical.

While the generally negative relationship between median and mean values of per element agreement and the level of communality is present in all factor extraction methods, the pattern is not as strongly expressed in distributions associated with Ordinary Least Squares. In all conditions, Ordinary Least Squares factor extraction method yields distributions that have smaller ranges and semi-interquartile ranges than the other two methods. Variability in this measure is positively related to communality for both Principal Axis and Maximum Likelihood methods; for Ordinary Least Squares, this positive relationship in variability is only present in the interactions that contain 60 observed variables. In most of the interactions, the mean per element agreement value is negatively related to the number of observed variables; this negative relationship is less pronounced with the Ordinary Least Squares factor extraction method and not present in the interaction that includes the wide communality range. As was the case with the loading sensitivity measure, variability in the distribution of per element agreement values was the smallest for the Ordinary Least Squares method across all interactions between the number of observed variables and level of communality.

Table 30

*Means and Standard Deviations of Per Element Agreement for Factor Extraction Methods and Number of Observed Variables by Community Range Interaction (P x H)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(P x H)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 20 × Low | .654 | 0.070 | .728 | 0.062 | .688 | 0.065 |
| 20 × Wide | .625 | 0.077 | .733 | 0.049 | .642 | 0.082 |
| 20 × High | .599 | 0.096 | .727 | 0.055 | .614 | 0.099 |
| 40 × Low | .624 | 0.176 | .752 | 0.075 | .638 | 0.184 |
| 40 × Wide | .581 | 0.193 | .763 | 0.067 | .588 | 0.196 |
| 40 × High | .520 | 0.232 | .744 | 0.090 | .525 | 0.235 |
| 60 × Low | .638 | 0.191 | .782 | 0.091 | .644 | 0.194 |
| 60 × Wide | .577 | 0.212 | .768 | 0.087 | .580 | 0.214 |
| 60 × High | .521 | 0.234 | .753 | 0.096 | .524 | 0.236 |

*n = 540*

The mean values of per element pattern agreement associated with the sample size by communality interaction (N × H) are presented in Table 31. For all levels of the interaction, Ordinary Least Squares yields the highest values of the per element factor agreement measure. As illustrated in Figure 9 (see Appendix B), the relationship between sample size and per element agreement is positive. For Maximum Likelihood and Principal Axis Factor extraction methods, the relationship between per element agreement and communality is negative.

Table 31
*Means and Standard Deviations of Per Element Agreement for Factor Extraction Method and Sample Size by Communality Range Interaction (N x H)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(N x H)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 100 × Low | .619 | 0.140 | .687 | 0.054 | .644 | 0.144 |
| 100 × Wide | .581 | 0.160 | .717 | 0.057 | .594 | 0.164 |
| 100 × High | .537 | 0.189 | .722 | 0.069 | .547 | 0.192 |
| 200 × Low | .636 | 0.153 | .745 | 0.066 | .656 | 0.159 |
| 200 × Wide | .593 | 0.171 | .752 | 0.066 | .603 | 0.176 |
| 200 × High | .546 | 0.201 | .741 | 0.081 | .554 | 0.204 |
| 300 × Low | .643 | 0.159 | .772 | 0.069 | .660 | 0.164 |
| 300 × Wide | .598 | 0.176 | .766 | 0.070 | .606 | 0.180 |
| 300 × High | .549 | 0.205 | .747 | 0.086 | .556 | 0.209 |
| 1000 × Low | .657 | 0.169 | .812 | 0.074 | .664 | 0.173. |
| 1000 × Wide | .605 | 0.184 | .784 | 0.076 | .611 | 0.187 |
| 1000 × High | .554 | 0.213 | .755 | 0.093 | .559 | 0.216 |

*N = 540*

Across all interactions among main effects, mean per element agreement values for the Ordinary Least Squares factor extraction exceeded the mean values associated with Principal Axis and Maximum Likelihood methods. For interactions that included the number of factors imbedded in the population main effect, the differences among the three factor extraction methods, in terms of mean per element agreement, diminished as the number of factors increased. In all interactions that included communality, the relationship between mean per element agreement values and communality was negative; this relationship was apparent for all factor extraction methods.

**Factor Loading Bias**

Table 32 provides a description of the univariate distribution of factor loading bias estimates for the three tested factor extraction methods. A Shapiro-Wilks' tests of normality yielded evidence of non-normality in the distributions of bias for all factor extraction methods. Mauchly's test of transformed variables $\chi^2$ (2) = 1544.3812, $p < .0001$, indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To address the potential for increased Type I error rate, tests of significance associated with the repeated measures analysis of variance will be based on the Greenhouse-Geisser adjustment to degrees of freedom (Stevens, 2002).

Table 32
*Descriptive Statistics for Distribution of Loading Bias Estimates*

| Factor Extraction Method | N | M | SD | Skewness | Kurtosis |
|---|---|---|---|---|---|
| Principal Axis | 540 | -0.121 | 0.064 | -1.016 | -0.084 |
| Ordinary Least Squares | 540 | 0.027 | 0.022 | -0.191 | -0.439 |
| Maximum Likelihood | 540 | -0.122 | 0.063 | -1.023 | -0.069 |

Results of the multivariate analysis of variance indicate that levels of factor loading bias differed significantly across the tested factor extraction methods, , $\Lambda = .0018$, $F$ (2, 459) = 127099, $p < .0001$ (see Table A5 in Appendix A); the multivariate analysis highlighted significant differences in mean bias by number of factors, number of observed variables, communality range, and level of dichotomization. The univariate, repeated measures analysis of variance also indicates that factor loading bias differ significantly by factor extraction method, $F(2, 920) = 25232$, $p < .0001$, $\eta_G^2 = .992$ (see Table A6 in appendix A). The model including main effects and first-order interactions accounted for 99.49% of the variability associated with statistical bias in factor pattern matrices.

In addition to the method main effects, the multivariate and repeated measures analyses of variance identified significant differences in bias associated with six of the ten first-order interactions. However, only four of these interactions yielded effect sizes that were medium or greater. These interactions included

1. Number of factors by observed variables, $F(4, 460) = 244.87$, $p < .0001$, $\eta_G^2 = .621$
2. Number of factors by sample size, $F(6, 460) = 15.62$, $p < .0001$, $\eta_G^2 = .136$
3. Number of factors by level of communality, $F(4, 460) = 454.50$, $p < .0001$, $\eta_G^2 = .753$
4. Number of factors by level of dichotomization, $F(8, 460) = 4.91$, $p < .0001$, $\eta_G^2 = .062$

Comparisons among means were conducted for these interactions only.

Table 33 presents mean factor loading biases by for each factor extraction method by each level of the interaction between number of factors and number of observed variables. For each level of the interaction, the comparisons indicate that mean bias values for Ordinary Least Squares are closer to zero than they are for either the Maximum Likelihood or the Principal Axis methods. Biases for Principal Axis and Maximum Likelihood are negative.

As both Table 33 and Figure 10 (see Appendix B) highlight, the absolute value of loading bias decrease as the number of factors increase; however, this relationship is more strongly expressed in the distributions associated with Principal Axis and Maximum Likelihood factor extraction methods than it is in distributions associated with Ordinary Least Squares. The box and whisker plots in Figure 10 indicate that the ranges and semi-interquartile ranges in distributions associated with Maximum Likelihood and Principal Axis diminish as the number of factors increase. In distributions of loading bias associated with the Ordinary Least Squares factor extraction method, the mean and median levels of bias are positively related to the ratio of observed variables to factors across all levels of the interaction.

Table 33
*Means and Standard Deviations of Factor Loading Bias for Factor Extraction Method and Number of Factors by Number of Observed Variables Interaction (K x P)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x P)* | M | SD | M | SD | M | SD |
| 2 × 20 | -.170 | 0.052 | .022 | 0.010 | -.171 | 0.053 |
| 2 × 40 | -.200 | 0.050 | .035 | 0.013 | -.200 | 0.051 |
| 2 × 60 | -.213 | 0.048 | .037 | 0.011 | -.213 | 0.048 |
| 4 × 20 | -.089 | 0.023 | .022 | 0.024 | -.090 | 0.023 |
| 4 × 40 | -.093 | 0.025 | .046 | 0.015 | -.094 | 0.025 |
| 4 × 60 | -.102 | 0.025 | .053 | 0.011 | -.101 | 0.026 |
| 8 × 20 | -.085 | 0.017 | -.007 | 0.009 | -.085 | 0.017 |
| 8 × 40 | -.073 | 0.017 | .011 | 0.014 | -.074 | 0.017 |
| 8 × 60 | -.065 | 0.015 | .026 | 0.013 | -.067 | 0.015 |

*n = 540*

Table 34 provides means and standard deviations for mean bias levels across factor extraction methods and levels of the number of factors by sample size interaction. As was the case with the previous set of comparisons, the absolute values of mean loading bias decrease as the number of factors increase.

Table 34
*Means and Standard Deviations of Factor Loading Bias by Factor Extraction Method and Number of Factors by Sample Size Interaction (K x N)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x N)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 2 × 100 | -.198 | 0.054 | .030 | 0.011 | -.200 | 0.064 |
| 2 × 200 | -.195 | 0.054 | .032 | 0.013 | -.200 | 0.054 |
| 2 × 300 | -.194 | 0.054 | .032 | 0.013 | -.194 | 0.054 |
| 2 × 1000 | -.192 | 0.054 | .033 | 0.016 | -.193 | 0.054 |
| 4 × 100 | -.103 | 0.022 | .029 | 0.024 | -.103 | 0.022 |
| 4 × 200 | -.095 | 0.024 | .040 | 0.021 | -.095 | 0.024 |
| 4 × 300 | -.093 | 0.025 | .043 | 0.020 | -.092 | 0.025 |
| 4 × 1000 | -.088 | 0.028 | .048 | 0.018 | -.089 | 0.027 |
| 8 × 100 | -.085 | 0.016 | -.002 | 0.015 | -.085 | 0.015 |
| 8 × 200 | -.076 | 0.016 | .008 | 0.017 | -.077 | 0.016 |
| 8 × 300 | -.072 | 0.017 | .013 | 0.017 | -.073 | 0.017 |
| 8 × 1000 | -.065 | 0.017 | .020 | 0.017 | -.066 | 0.017 |

*N = 540*

At the eight-factor by N = 100 condition, Ordinary Least Squares yields a mean bias of -.002; this is the smallest average amount of bias across all factor methods and all interactions between number of factors and sample size.

As the box and whisker plots in Figure 11 (see Appendix B) illustrate, Maximum Likelihood and Principal Axis Factor extraction methods yield factor pattern matrices that exhibit similar patterns of negative bias. In distributions associated with Maximum Likelihood and Principal Axis Factor extraction methods, the mean and median levels of bias decrease (in absolute value) as the number of factors and the sample sizes increase; ranges and semi-interquartile ranges, for these distributions, are also negatively related to both number of factors and sample size.

Table 35 provides means and standard deviations of loading bias estimates for the three factor extraction methods for the number of factors by communality range interaction. Ordinary Least Squares yields factor loading bias estimates that are closer to zero than the other two factor extraction methods. For Maximum Likelihood and Principal Axis Factor extraction methods, all mean levels of factor loading bias are negative. In terms of absolute value, the smallest mean level of factor loading bias is associated with the eight-factor by low communality condition ($<.001$ and $> 0$).

In the distributions of bias associated with Maximum Likelihood and Principal Axis Factor extraction methods, the box and whisker plots (see Figure 12 in Appendix B) indicate that the absolute values of bias are positively related to communality and negatively related to the number of factors imbedded in the population. This pattern is not expressed in the distribution of bias values for Ordinary Least Squares. For Maximum Likelihood and Principal Axis methods, the mean and median diminish in absolute value as the number of actors increase; this pattern is not expressed in the distribution of bias values for Ordinary Least Squares.

Table 35
*Means and Standard Deviations of Factor Loading Bias for Factor Extraction Method and Number of Factors by Communality Range Interaction (K x H)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x H)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 2 × Low | -.132 | 0.021 | .027 | 0.010 | -.132 | 0.022 |
| 2 × Wide | -.200 | 0.018 | .030 | 0.010 | -.200 | 0.018 |
| 2 × High | -.253 | 0.019 | .037 | 0.017 | -.253 | 0.019 |
| 4 × Low | -.065 | 0.009 | .025 | 0.020 | -.066 | 0.009 |
| 4 × Wide | -.097 | 0.009 | .039 | 0.019 | -.098 | 0.009 |
| 4 × High | -.121 | 0.011 | .056 | 0.014 | -.121 | 0.011 |
| 8 × Low | -.058 | 0.011 | <.001 | 0.017 | -.058 | 0.011 |
| 8 × Wide | -.073 | 0.010 | .011 | 0.015 | -.075 | 0.009 |
| 8 × High | -.092 | 0.013 | .019 | 0.018 | -.092 | 0.013 |

*N = 540*

Table 36 includes means and standard deviations of factor loading bias values for each level of the interaction between number of factors and level of dichotomization interaction. The mean biase values for Principal Axis and Maximum Likelihood factor extraction methods are negative for each level of the interaction. Across all levels of the interaction, Ordinary Least Squares resulted in the smallest mean amount of factor loading bias.

Table 36
*Means and Standard Deviations of Factor Loading Bias for Factor Extraction Method and Number of Factors by Level of Dichotomization Interaction (K x D)*

| | Principal Axis | | Ordinary Least Squares | | Maximum Likelihood | |
|---|---|---|---|---|---|---|
| Interaction *(K x D)* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| 2 × .05 | -.194 | 0.059 | .033 | 0.012 | -.194 | 0.059 |
| 2 × .25 | -.194 | 0.055 | .031 | 0.014 | -.194 | 0.055 |
| 2 × .50 | -.190 | 0.050 | .038 | 0.014 | -.190 | 0.050 |
| 2 × .75 | -.197 | 0.053 | .024 | 0.013 | -.197 | 0.052 |
| 2 × .95 | -.200 | 0.053 | .031 | 0.011 | -.200 | 0.053 |
| 4 × .05 | -.096 | 0.025 | .040 | 0.022 | -.096 | 0.025 |
| 4 × .25 | -.092 | 0.025 | .043 | 0.022 | -.093 | 0.025 |
| 4 × .50 | -.093 | 0.025 | .041 | 0.023 | -.094 | 0.025 |
| 4 × .75 | -.094 | 0.025 | .040 | 0.020 | -.095 | 0.025 |
| 4 × .95 | -.097 | 0.026 | .038 | 0.023 | -.098 | 0.026 |
| 8 × .05 | -.073 | 0.017 | .011 | 0.017 | -.074 | 0.017 |
| 8 × .25 | -.073 | 0.018 | .012 | 0.019 | -.073 | 0.018 |
| 8 × .50 | -.074 | 0.018 | .010 | 0.019 | -.075 | 0.018 |
| 8 × .75 | -.075 | 0.018 | .009 | 0.017 | -.075 | 0.018 |
| 8 × .95 | -.078 | 0.019 | .007 | 0.020 | -.079 | 0.018 |

*N = 540*

The box and whisker plots (see Figure 13 in appendix B) indicate that the absolute values of means and median levels of statistical bias in the factor loading matrices diminish as the number of factors increase. For Maximum Likelihood and Principal Axis Factor extraction methods, the ranges and semi-interquartile ranges also diminish as the number of factors increase. These patterns in measures of central tendency and dispersion are not apparent in the distributions of loading bias associated with the Ordinary Least Squares factor extraction method.

## Conclusions

Across the majority of interactions among the manipulated research contexts that accounted for statistically significant differences and moderate effect sizes, the Ordinary Least Squares factor extraction method yielded factor loading matrices that were in better agreement with the population than either the Maximum Likelihood or the Principal Axis methods. The Ordinary Least Squares method yielded factor loading matrices that exhibited less bias and resolved existing factors at a better rate than the other two tested factor extraction methods.

In the multivariate and repeated measures of analyses, level dichotomization, as a main effect, contributed to significant differences in factor loading sensitivity, per element agreement, and factor loading bias. In 58% of the first-order interactions that incorporated level of dichotomization as an effect, the analyses yielded significant differences in the outcome variables. However, only two of these interactions yielded differences of moderate effect size or greater; these interactions were associated with factor loading sensitivity and factor loading bias.

The number of factors effect was included in both cases in which level of dichotomization contributed to differences in the outcome variables of at least moderate in effect size. As the Box plot in figures 3 and 13 (See Appendix B) illustrate, the influence of the number of factors effect overwhelmed the relationship between dichotomization and the outcome variables: factor loading sensitivity and factor loading bias. As noted earlier, Ordinary Least Squares factor analysis yielded factor loading matrices that were in better agreement with the known population in both interactions that included dichotomizations.

The analyses included in this study focused on all first-order interactions among five manipulated research characteristic. Results of these analyses led to the following conclusions concerning these manipulated research characteristics:

1. As the number of factors in the population increases, the factor loading values become more representative of the population parameters. This conclusion is well supported by the measures of factor loading sensitivity and loading bias.
2. Increased ratios of observed variables to factors are not necessarily associated with improved agreement between sample and the population factor loading matrices. This conclusion appears to contradict previous methodological research associated with over-determination (Fabrigar et al., 1999; Guadagnoli & Velicer, 1988; Hogarty et al., 2005).
3. In terms of sample agreement with the population, the results associated with interactions that include communality are contradictory. For all factor extraction methods included in this study, factor loading sensitivity tends to improve as the level of communality increases. However, per element agreement diminishes as the level of communality increases; moreover, the amount of factor loading bias also appears to be positively associated with the level of communality.

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## Appendix A

## Multivariate Analyses of Variance Summary Tables

Table A1

*Multivariate Analysis of Variance Summary for Factor Loading Sensitivity (OK_LOAD)Values*

| | Df | Λ | f | P > f |
|---|---|---|---|---|
| Factor Extraction Method (M) | 2 (459) | .0001 | 342901* | <.0001 |
| M × No. of Factors (K) | 4 (918) | .0026 | 4277.32* | <.0001 |
| M × No. Observed Variables (P) | 4 (918) | .0024 | 4454.70* | <.0001 |
| M × Sample Size (N) | 6 (918) | .8863 | 9.51* | <.0001 |
| M × Communality Level (H) | 4 (918) | .5980 | 67.26* | <.0001 |
| M × Dichotomization (D) | 8 (918) | .8226 | 11.77* | <.0001 |
| M × K × P | 8 (918) | .0270 | 582.84* | <.0001 |
| M × K × N | 12 (918) | .8134 | 8.32* | <.0001 |
| M × K × H | 8 (918) | .6907 | 22.32* | <.0001 |
| M × K × D | 16 (918) | .8521 | 4.78* | <.0001 |
| M × P × N | 12 (918) | .8775 | 5.16* | <.0001 |
| M × P × H | 8 (918) | .6109 | 32.06* | <.0001 |
| M × P × D | 16 (918) | .9381 | 1.86* | .0204 |
| M × N × H | 12 (918) | .9735 | 1.04 | .4134 |
| M × N × D | 24 (918) | .9856 | 0.28 | .9998 |
| M × H × D | 16 (918) | .9134 | 2.66* | .0004 |

* Significant at alpha = .025 level

Table A2

*Repeated Measures Analysis of Variance for Factor Loading Sensitivity (OK Load) Values*

| Source | df | SS | MS | F | P | $\eta_G^2$ |
|---|---|---|---|---|---|---|
| | | | Between Subjects | | | |
| Number of Factors (K) | 2 | 26.477 | 13.239 | 34780.8* | <.0001 | .999 |
| Number of Variables (P) | 2 | 41.680 | 20.840 | 54751.0* | <.0001 | .993 |
| Sample Size (N) | 3 | 0.012 | 0.004 | 10.96* | <.0001 | .044 |
| Communality (H) | 2 | 0.142 | 0.071 | 186.45* | <.0001 | .343 |
| Dichotomization (D) | 4 | 0.027 | 0.007 | 17.86* | <.0001 | .091 |
| K × P | 4 | 1.693 | 0.423 | 1111.93* | <.0001 | .862 |
| K × N | 6 | 0.003 | 0.0005 | 1.42 | .205 | .011 |
| K × H | 4 | 0.018 | 0.004 | 11.81* | <.0001 | .062 |
| K × D | 8 | 0.016 | 0.002 | 5.43* | <.0001 | .574 |
| P × N | 6 | 0.003 | 0.0006 | 1.51 | .174 | .012 |
| P × H | 4 | 0.216 | 0.005 | 14.21* | <.0001 | .074 |
| P × D | 8 | 0.004 | 0.0005 | 1.38 | .204 | .015 |
| N × H | 6 | 0.001 | 0.0002 | 0.55 | .770 | .004 |
| N × D | 12 | 0.002 | 0.0001 | 0.38 | .969 | .006 |
| H × D | 8 | 0.10 | 0.001 | 3.41* | .001 | .037 |
| Error | 460 | 0.175 | 0.0003 | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| Within Subjects (with Greenhouse-Geisser asjusted Pr > F) | | | | | | |
| Factor Extraction Method (M) | 2 | 125.806 | 62.903 | 600815* | <.0001 | .998 |
| M × K | 4 | 13.599 | 3.400 | 32474.6* | <.0001 | .980 |
| M × P | 4 | 20.986 | 5.246 | 50111.1* | <.0001 | .987 |
| M × N | 6 | 0.003 | 0.0005 | 4.42* | .0019 | .010 |
| M × H | 4 | 0.018 | 0.005 | 44.39* | <.0001 | .064 |
| M × D | 8 | 0.014 | 0.0018 | 16.81* | <.0001 | .049 |
| M × K × P | 8 | 0.910 | 0.114 | 1086.34* | <.0001 | .770 |
| M × K × N | 12 | 0.009 | 0.0008 | 7.48* | <.0001 | .033 |
| M × K × H | 8 | 0.011 | 0.0014 | 13.09* | <.0001 | .038 |
| M × K × D | 16 | 0.010 | 0.0006 | 5.75* | <.0001 | .034 |
| M × P × N | 12 | 0.003 | 0.0002 | 2.21 | .0269 | .010 |
| M × P × H | 8 | 0.0171 | 0.0020 | 20.47* | <.0001 | .059 |
| M × P × D | 16 | 0.003 | 0.0002 | 1.74 | .0663 | .011 |
| M × N × H | 12 | 0.001 | 0.0001 | 1.16 | .325 | .005 |
| M × N × D | 24 | 0.001 | 0.0000 | 0.40 | .981 | .004 |
| M × H × D | 16 | 0.006 | 0.0004 | 3.61* | <.0001 | .022 |
| Error (Method) | 920 | 0.096 | 0.0001 | | | |

* Significant at the alpha = .025 level

Table A3

*Multivariate Analysis of Variance for Per Element Pattern Agreement (Pattern Accuracy)*

| | Df | Λ | f | P > f |
|---|---|---|---|---|
| Factor Extraction Method (M) | 2 (459) | .0085 | 26828.4* | <.0001 |
| M × No. of Factors (K) | 4 (918) | .0157 | 1599.46* | <.0001 |
| M × No. Observed Variables (P) | 4 (918) | .0888 | 540.51* | <.0001 |
| M × Sample Size (N) | 6 (918) | .3154 | 119.42* | <.0001 |
| M × Communality Level (H) | 4 (918) | .1318 | 402.65* | <.0001 |
| M × Dichotomization (D) | 8 (918) | .9611 | 2.30* | .0192 |
| M × K × P | 8 (918) | .1220 | 213.70* | <.0001 |
| M × K × N | 12 (918) | .7942 | 9.34* | <.0001 |
| M × K × H | 8 (918) | .6730 | 25.12* | <.0001 |
| M × K × D | 16 (918) | .8355 | 5.39* | <.0001 |
| M × P × N | 12 (918) | .8618 | 5.91* | <.0001 |
| M × P × H | 8 (918) | .6102 | 32.14* | <.0001 |
| M × P × D | 16 (918) | .9431 | 1.71 | .0403 |
| M × N × H | 12 (918) | .5823 | 23.75* | <.0001 |
| M × N × D | 24 (918) | .9940 | 0.12 | 1.0000 |
| M × H × D | 16 (918) | 0.9336 | 2.00* | .0107 |

* Significant at alpha = .025 level

Table A4

*Repeated Measures Analysis of Variance for Per Element Pattern Agreement (Pattern Accuracy) Values*

| Source | Df | SS | MS | F | P | $\eta_G^2$ |
|---|---|---|---|---|---|---|
| | | | Between Subjects | | | |
| Number of Factors (K) | 2 | 26.148 | 13.074 | 12137.8* | <.0001 | .978 |
| Number of Variables (P) | 2 | 0.285 | 0.142 | 132.18* | <.0001 | .330 |
| Sample Size (N) | 3 | 0.338 | 0.113 | 104.67* | <.0001 | .370 |
| Communality (H) | 2 | 1.293 | 0.646 | 600.08* | <.0001 | .691 |
| Dichotomization (D) | 4 | 0.017 | 0.004 | 4.01* | .0033 | .029 |
| K × P | 4 | 3.874 | 0.968 | 899.10* | <.0001 | .870 |
| K × N | 6 | 0.120 | 0.020 | 18.540* | <.0001 | .172 |
| K × H | 4 | 0.417 | 0.104 | 96.70* | <.0001 | .419 |
| K × D | 8 | 0.035 | 0.004 | 4.07* | .0001 | .057 |
| P × N | 6 | 0.027 | 0.004 | 4.16* | .0004 | .044 |
| P × H | 4 | 0.106 | 0.026 | 25.54* | <.0001 | .155 |
| P × D | 8 | 0.030 | 0.004 | 3.43* | .0007 | .049 |
| N × H | 6 | 0.059 | 0.010 | 9.10* | <.0001 | .092 |
| N × D | 12 | 0.001 | 0.000 | 0.05 | 1.000 | .001 |
| H × D | 8 | 0.020 | 0.002 | 2.33* | 0.019 | .033 |
| Error | 460 | 0.495 | 0.001 | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| Within Subjects (with Greenhouse-Geisser asjusted Pr > F) | | | | | | |
| Factor Extraction Method | | | | | | |
| (M) | 2 | 8.235 | 4.118 | 45801.9* | <.0001 | .934 |
| M × K | 4 | 4.122 | 1.030 | 11461.9* | <.0001 | .877 |
| M × P | 4 | 0.641 | 0.160 | 1781.58* | <.0001 | .526 |
| M × N | 6 | 0.140 | 0.233 | 259.35* | <.0001 | .195 |
| M × H | 4 | 0.438 | 0.109 | 1219.6* | <.0001 | .431 |
| M × D | 8 | 0.002 | 0.000 | 2.86* | .0178 | .003 |
| M × K × P | 8 | 0.521 | 0.065 | 724.05* | <.0001 | .474 |
| M × K × N | 12 | 0.003 | 0.000 | 2.67* | .0100 | .004 |
| M × K × H | 8 | 0.028 | 0.003 | 38.25* | <.0001 | .045 |
| M × K × D | 16 | 0.121 | 0.001 | 8.43* | <.0001 | .020 |
| M × P × N | 12 | 0.002 | 0.000 | 1.96 | .0622 | .004 |
| M × P × H | 8 | 0.018 | 0.002 | 24.36* | <.0001 | .029 |
| M × P × D | 16 | 0.003 | 0.000 | 2.14* | .0226 | .005 |
| M × N × H | 12 | 0.048 | 0.004 | 44.65* | <.0001 | .077 |
| M × N × D | 24 | 0.000 | 0.000 | 0.09 | 1.0000 | .000 |
| M × H × D | 16 | 0.004 | 0.000 | 3.13* | .0009 | .007 |
| Error (Method) | 920 | 0.083 | 0.000 | | | |

* Significant at the alpha = .025 level

Table A5

*Multivariate Analysis of Variance for Estimates of Loading Bias (Bias Loadings)*

| | df | Λ | F | P > f |
|---|---|---|---|---|
| Factor Extraction Method (M) | 2 (459) | .0018 | 127099* | <.0001 |
| M × No. of Factors (K) | 4 (918) | .0114 | 1923.23* | <.0001 |
| M × No. Observed Variables (P) | 4 (918) | .1339 | 397.61* | <.0001 |
| M × Sample Size (N) | 6 (918) | .9765 | 1.83 | .0900 |
| M × Communality Level (H) | 4 (918) | .0182 | 1469.72* | <.0001 |
| M × Dichotomization (D) | 8 (918) | .9366 | 3.82* | .0002 |
| M × K × P | 8 (918) | .3271 | 85.88* | <.0001 |
| M × K × N | 12 (918) | .9207 | 3.22* | .0001 |
| M × K × H | 8 (918) | .1148 | 223.97* | <.0001 |
| M × K × D | 16 (918) | .9424 | 1.73 | .0369 |
| M × P × N | 12 (918) | .9775 | 0.88 | .5709 |
| M × P × H | 8 (918) | .8786 | 7.67* | <.0001 |
| M × P × D | 16 (918) | .9147 | 2.67* | .0005 |
| M × N × H | 12 (918) | .9888 | 0.43 | .9516 |
| M × N × D | 24 (918) | .9786 | 0.42 | .9944 |
| M × H × D | 16 (918) | .9116 | 2.72* | .0003 |

* Significant at alpha = .025 level

Table A6

*Repeated Measures Analysis of Variance for Estimates of Loading Bias (Bias Loadings)*

| Source | df | SS | MS | F | P | $\eta_G^2$ |
|---|---|---|---|---|---|---|
| | | | Between Subjects | | | |
| Number of Factors (K) | 2 | 1.826 | 0.913 | 8596.94* | <.0001 | .966 |
| Number of Variables (P) | 2 | < 0.001 | < 0.001 | 1.48 | .2290 | .005 |
| Sample Size (N) | 3 | 0.039 | 0.013 | 122.94* | <.0001 | .382 |
| Communality (H) | 2 | 0.441 | 0.220 | 2076.21* | <.0001 | .874 |
| Dichotomization (D) | 4 | 0.006 | 0.002 | 15.63* | <.0001 | .095 |
| K × P | 4 | 0.104 | 0.026 | 244.87* | <.0001 | .621 |
| K × N | 6 | 0.010 | 0.002 | 15.62* | <.0001 | .136 |
| K × H | 4 | 0.193 | 0.048 | 454.50* | <.0001 | .753 |
| K × D | 8 | 0.004 | < 0.001 | 4.91* | <.0001 | .062 |
| P × N | 6 | < 0.001 | < 0.001 | 0.85 | .5323 | .008 |
| P × H | 4 | < 0.001 | < 0.001 | 0.61 | .6567 | .004 |
| P × D | 8 | 0.002 | < 0.001 | 2.03 | .0410 | .026 |
| N × H | 6 | 0.001 | < 0.001 | 1.74 | .1098 | .017 |
| N × D | 12 | < 0.001 | < 0.001 | 0.09 | 1.000 | .002 |
| H × D | 8 | 0.003 | < 0.001 | 3.58* | .0005 | .046 |
| Error | 460 | 0.049 | < 0.001 | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| Within Subjects (with Greenhouse-Geisser asjusted Pr > F) | | | | | | |
| Factor Extraction Method (M) | 2 | 7.964 | 3.982 | 252532* | <.0001 | .992 |
| M × K | 4 | 1.237 | 0.309 | 19610.8* | <.0001 | .951 |
| M × P | 4 | .0925 | 0.231 | 1466.67* | <.0001 | .594 |
| M × N | 6 | .0001 | < 0.001 | 1.64 | .1778 | .002 |
| M × H | 4 | 0.492 | 0.123 | 7807.54* | <.0001 | .886 |
| M × D | 8 | < 0.001 | < 0.001 | 4.61* | .0011 | .009 |
| M × K × P | 8 | 0.022 | 0.003 | 176.98* | <.0001 | .261 |
| M × K × N | 12 | < 0.001 | < 0.001 | 1.98 | .0666 | .006 |
| M × K × H | 8 | 0.063 | 0.008 | 501.09* | <.0001 | .499 |
| M × K × D | 16 | 0.001 | < 0.001 | 2.38* | .0157 | .009 |
| M × P × N | 12 | < 0.001 | < 0.001 | 0.36 | .9033 | .001 |
| M × P × H | 8 | <0.001 | <0.001 | 4.01* | .0032 | .008 |
| M × P × D | 16 | 0.001 | < 0.001 | 3.40* | .0008 | .013 |
| M × N × H | 12 | < 0.001 | < 0.001 | 0.42 | .8693 | .001 |
| M × N × D | 24 | < 0.001 | < 0.001 | 0.17 | .9994 | .001 |
| M × H × D | 16 | < 0.001 | < 0.001 | 1.85 | .0644 | .007 |
| Error (Method) | 920 | 0.014 | < 0.001 | | | |

* Significant at the alpha = .025 level

99.4% of variance explained

# Appendix B

## Box and Whisker Plots

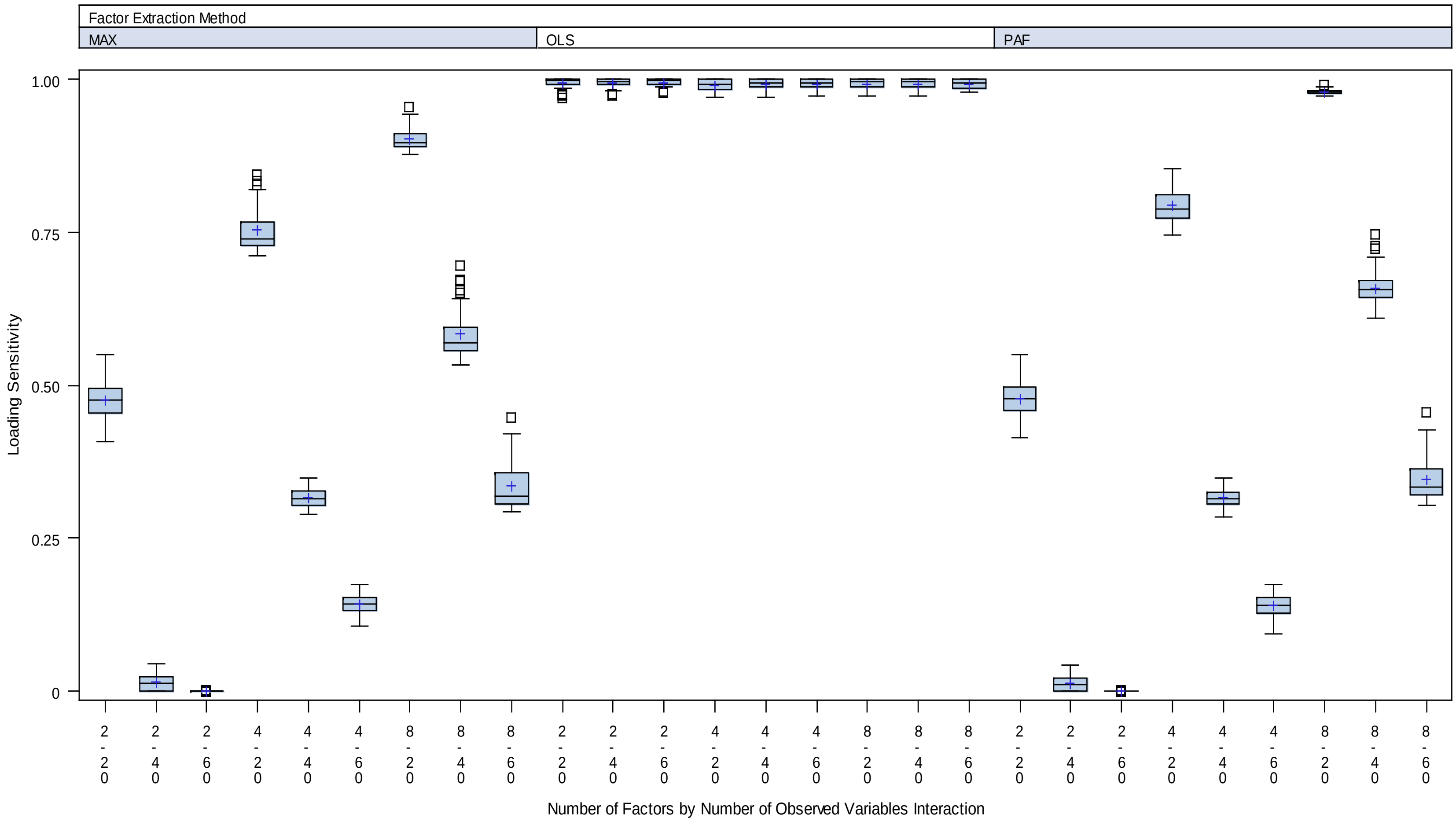


*Figure 1.* Factor loading sensitivity by the interaction between the number of factors and number of observed variables

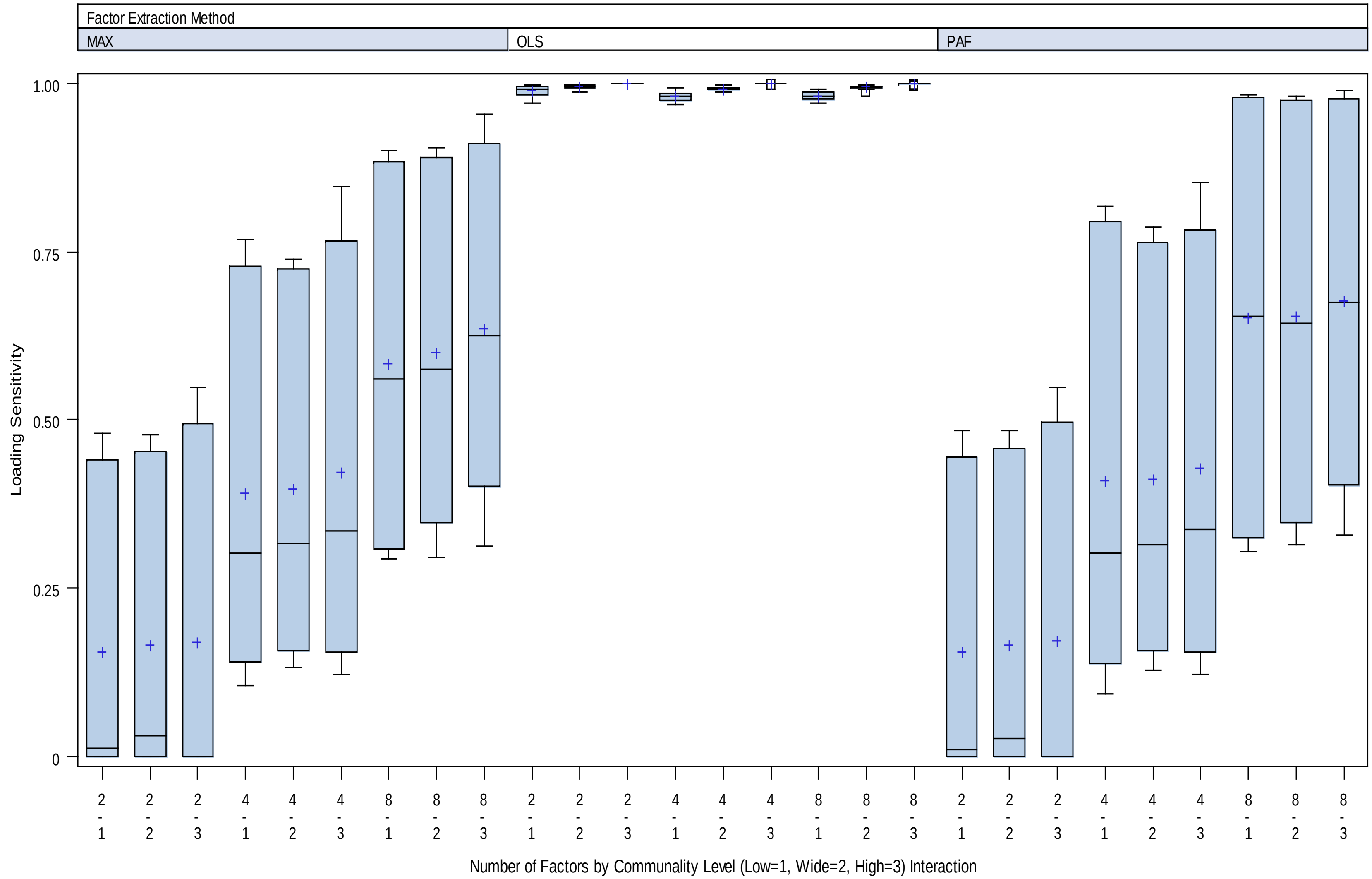


*Figure 2.* Factor loading sensitivity by the interaction between the number of factors and communality level

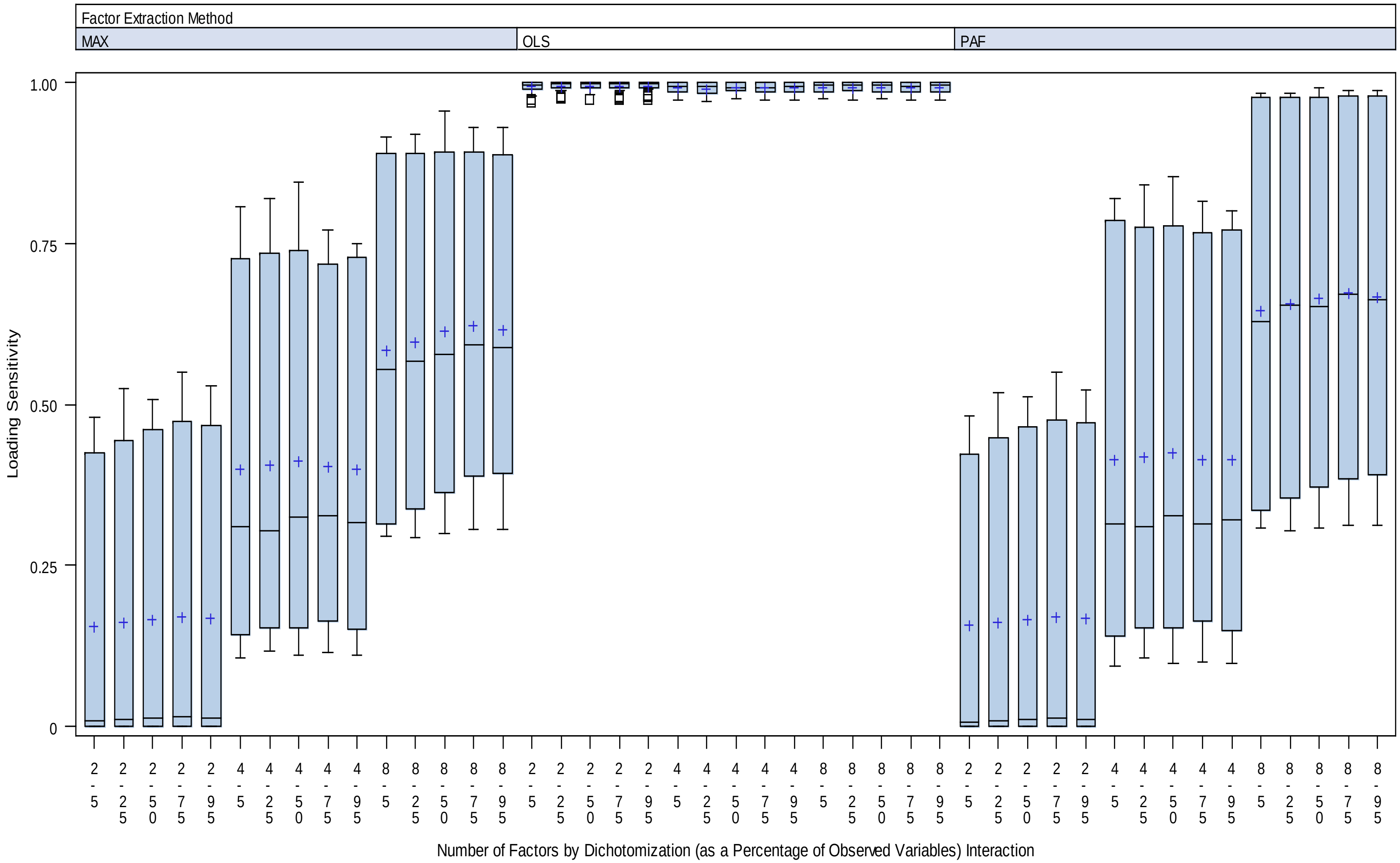


*Figure 3.* Factor loading sensitivity by the interaction between the number of factors and dichotomization

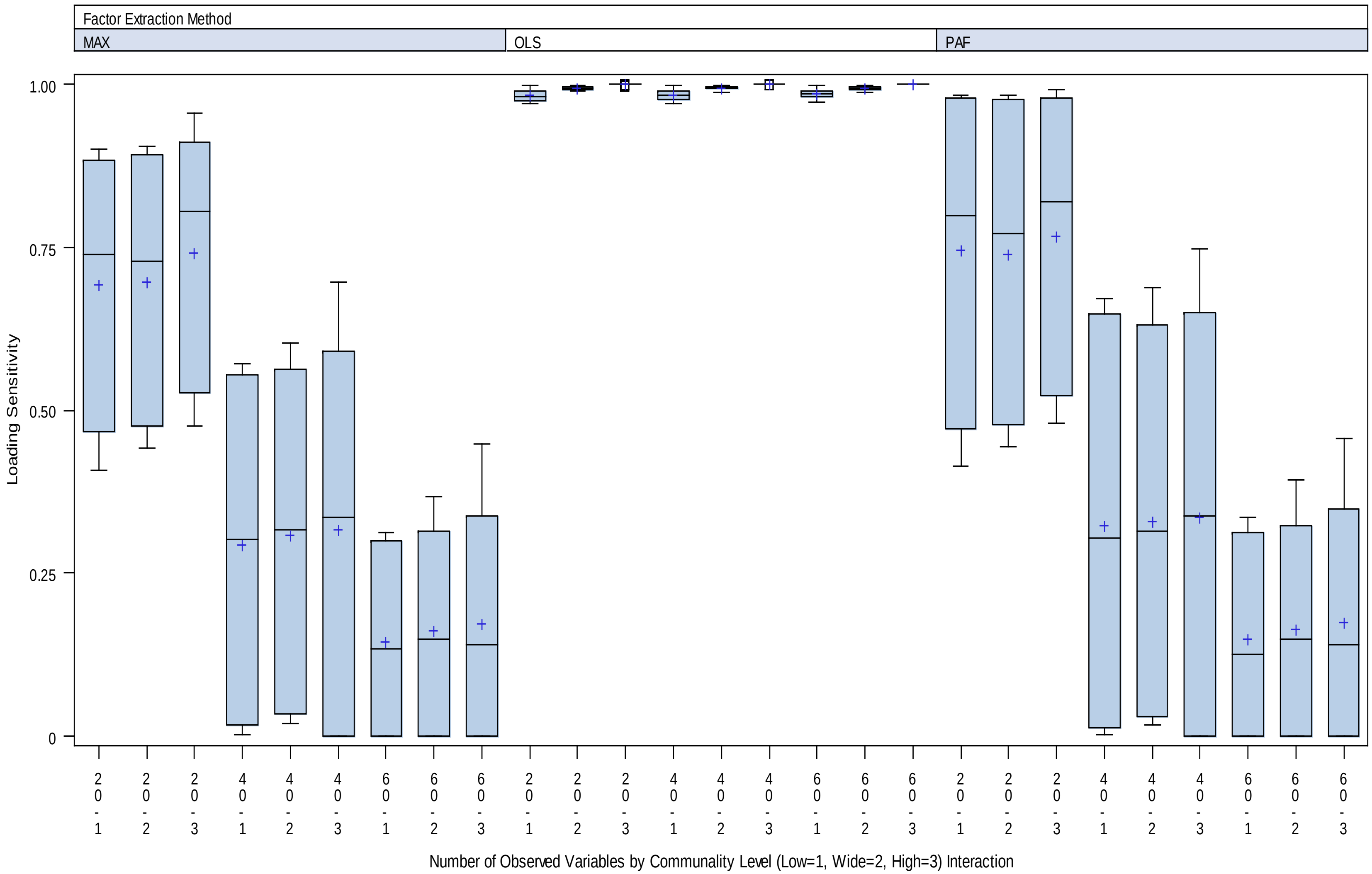


*Figure 4.* Factor loading sensitivity by the interaction between the number of observed variables by communality level

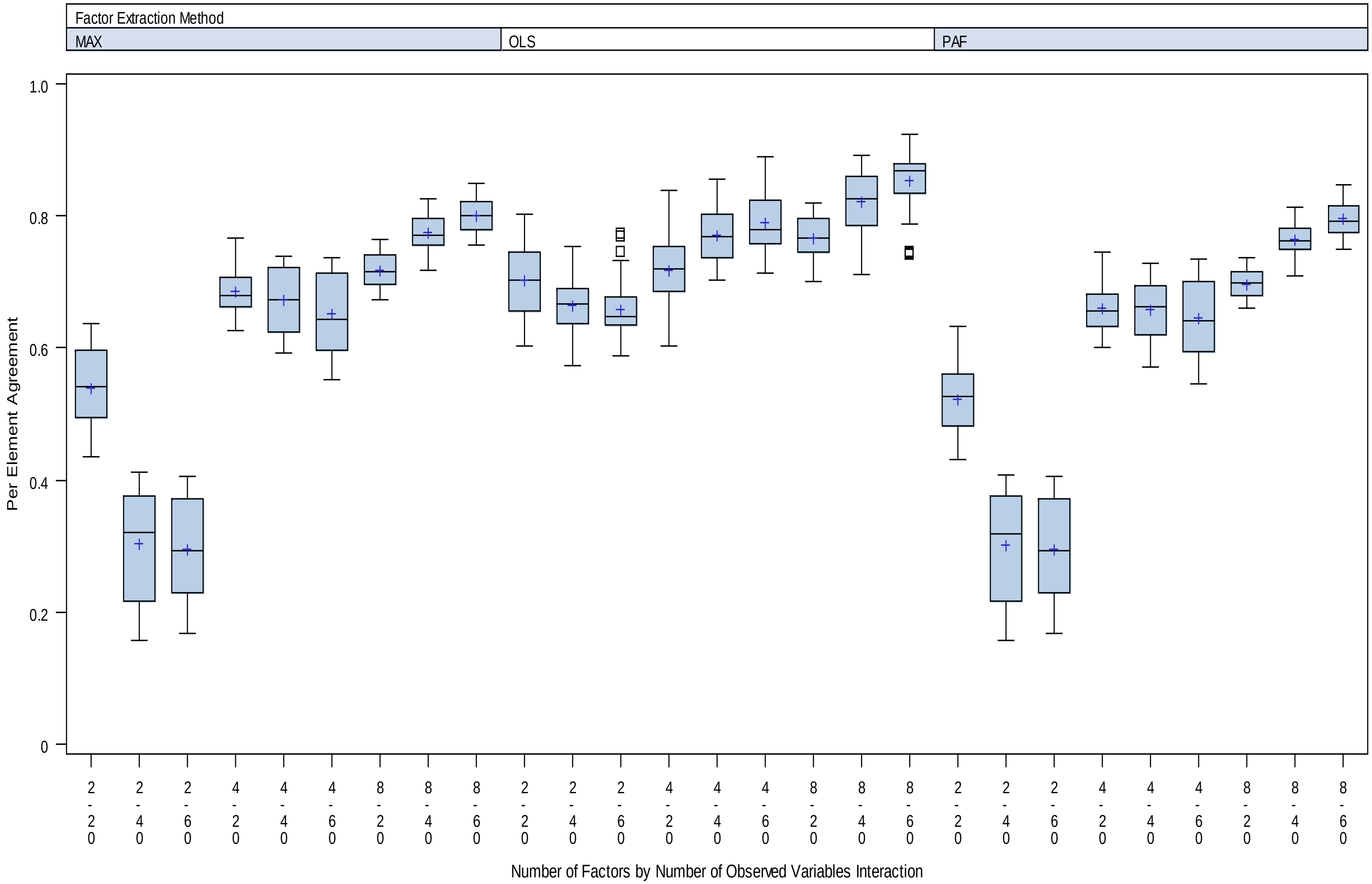


*Figure 5.* Per element factor pattern agreement by the interaction between number of factors and number of observed variables

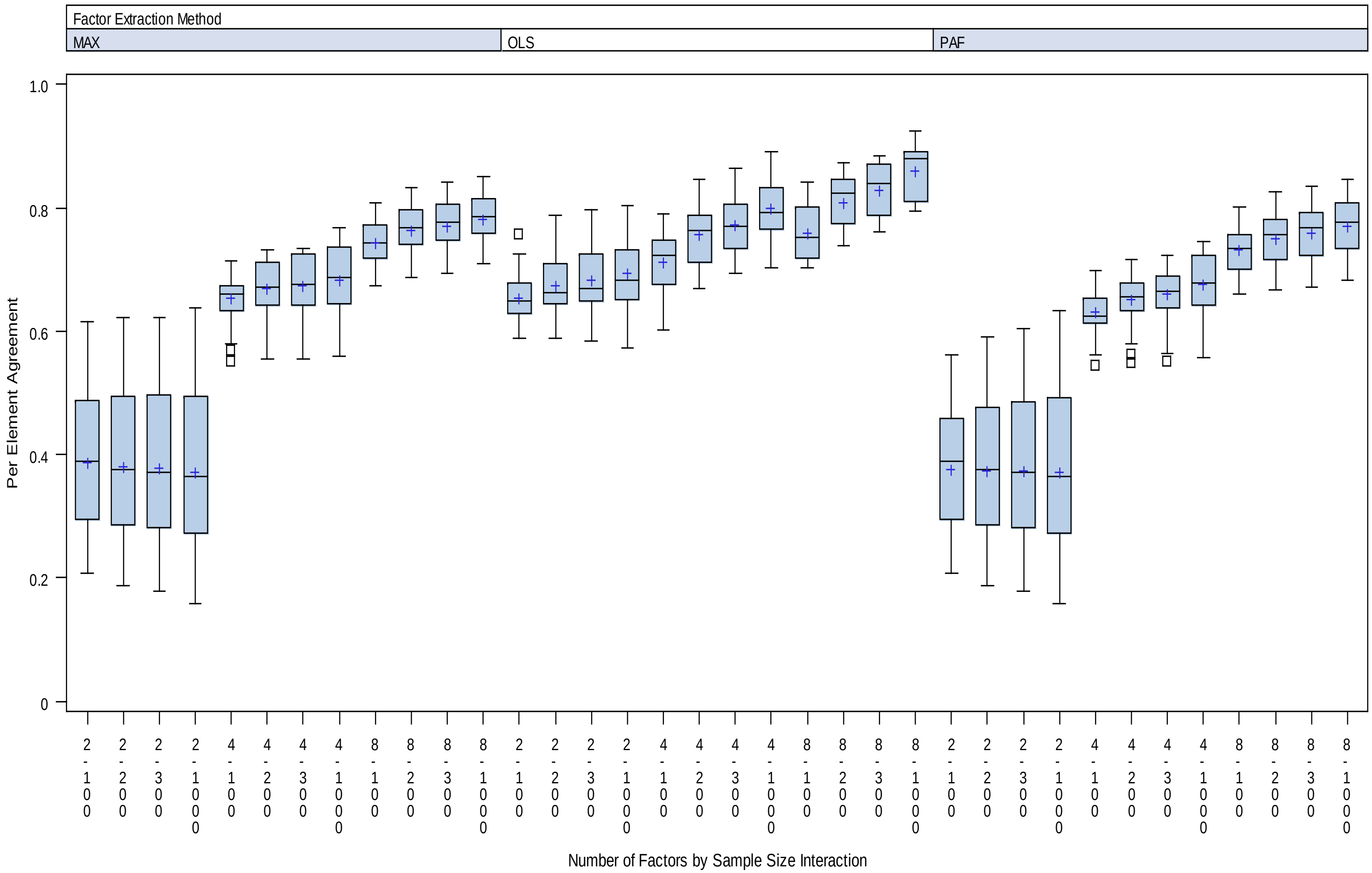


*Figure 6.* Per element factor pattern agreement by the interaction between number of factors and sample size

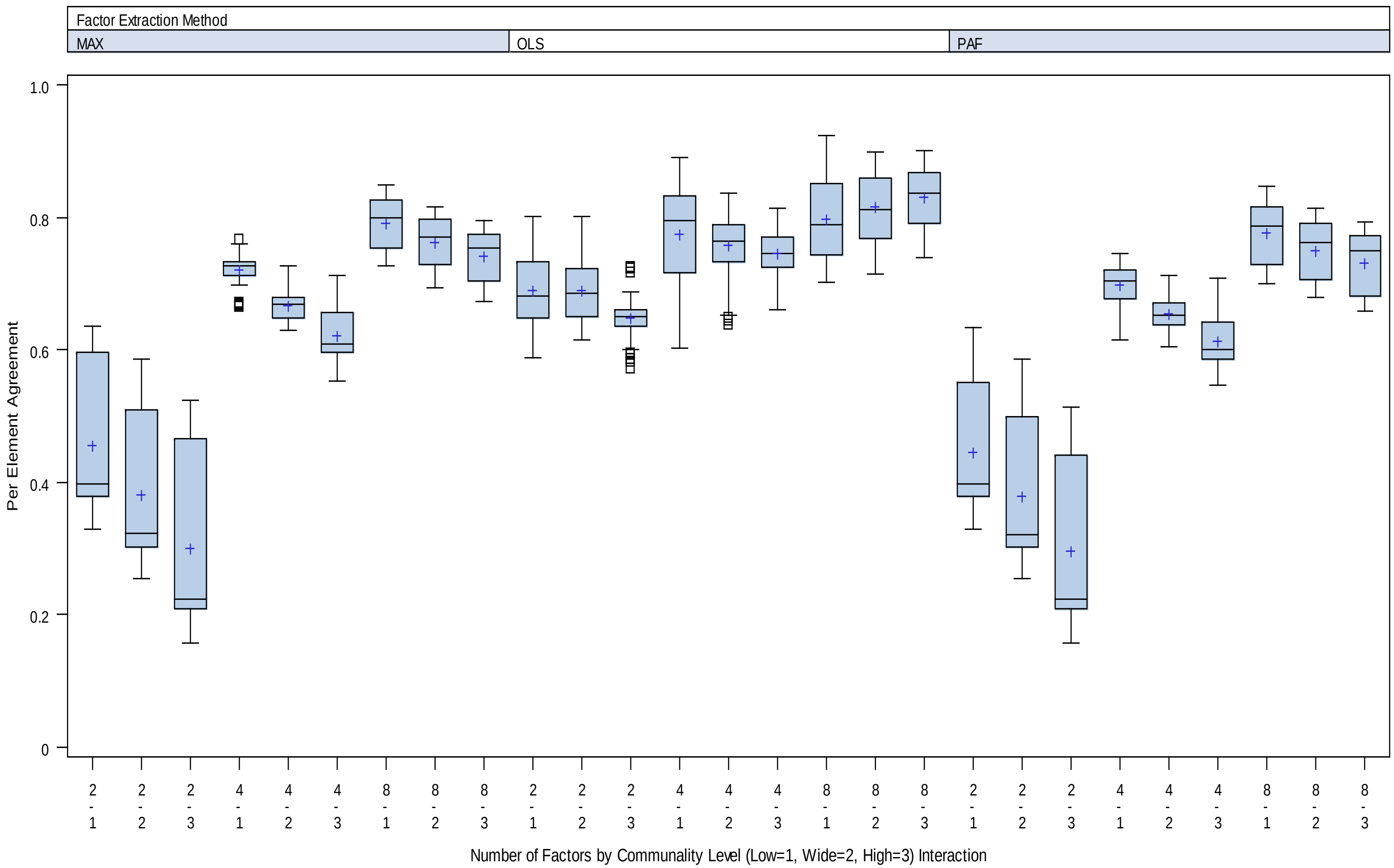


*Figure 7.* Per element factor pattern agreement by the interaction between number of factors and level of communality

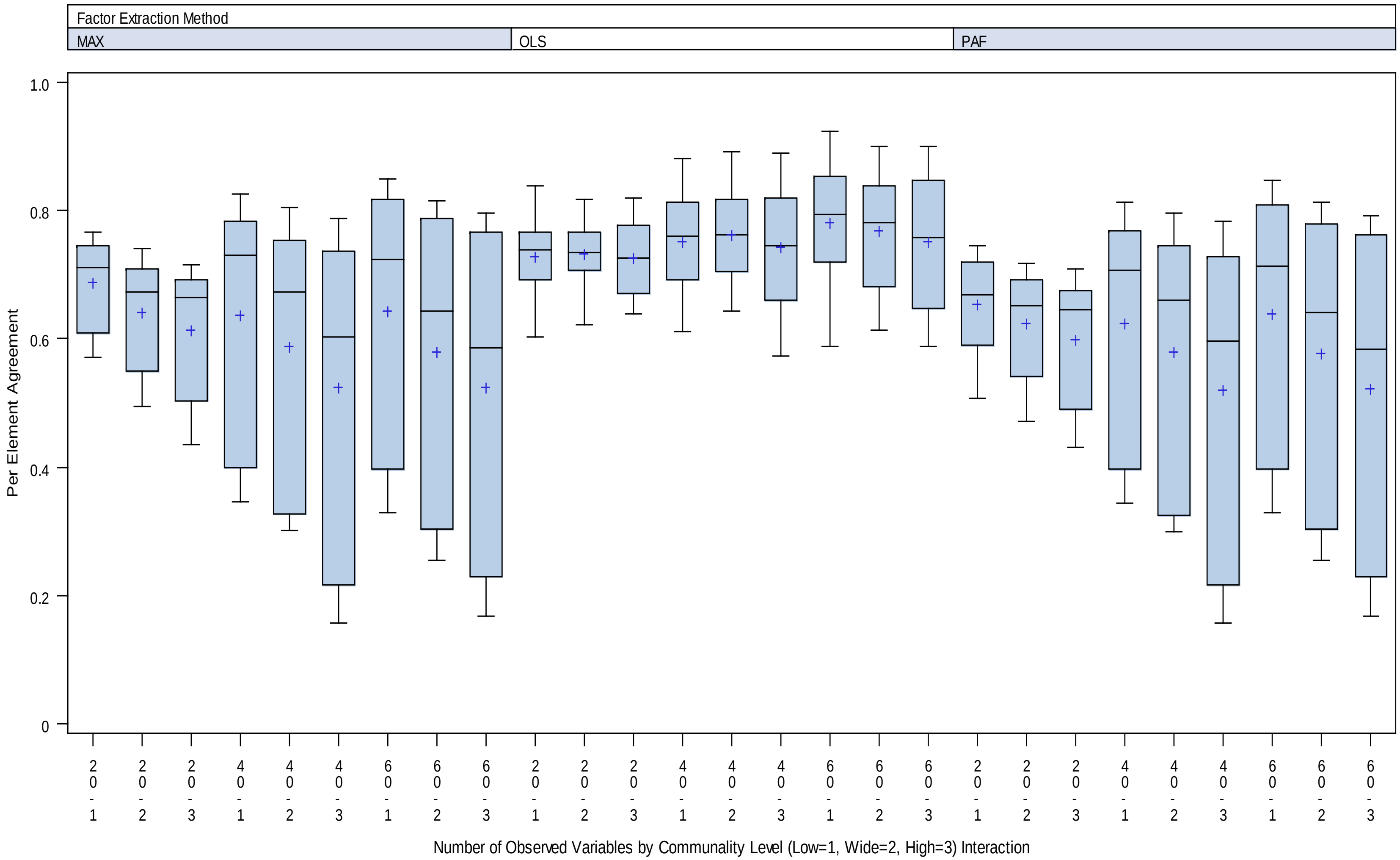


*Figure 8.* Per element factor pattern agreement by the interaction between number of observed variables and level of communality

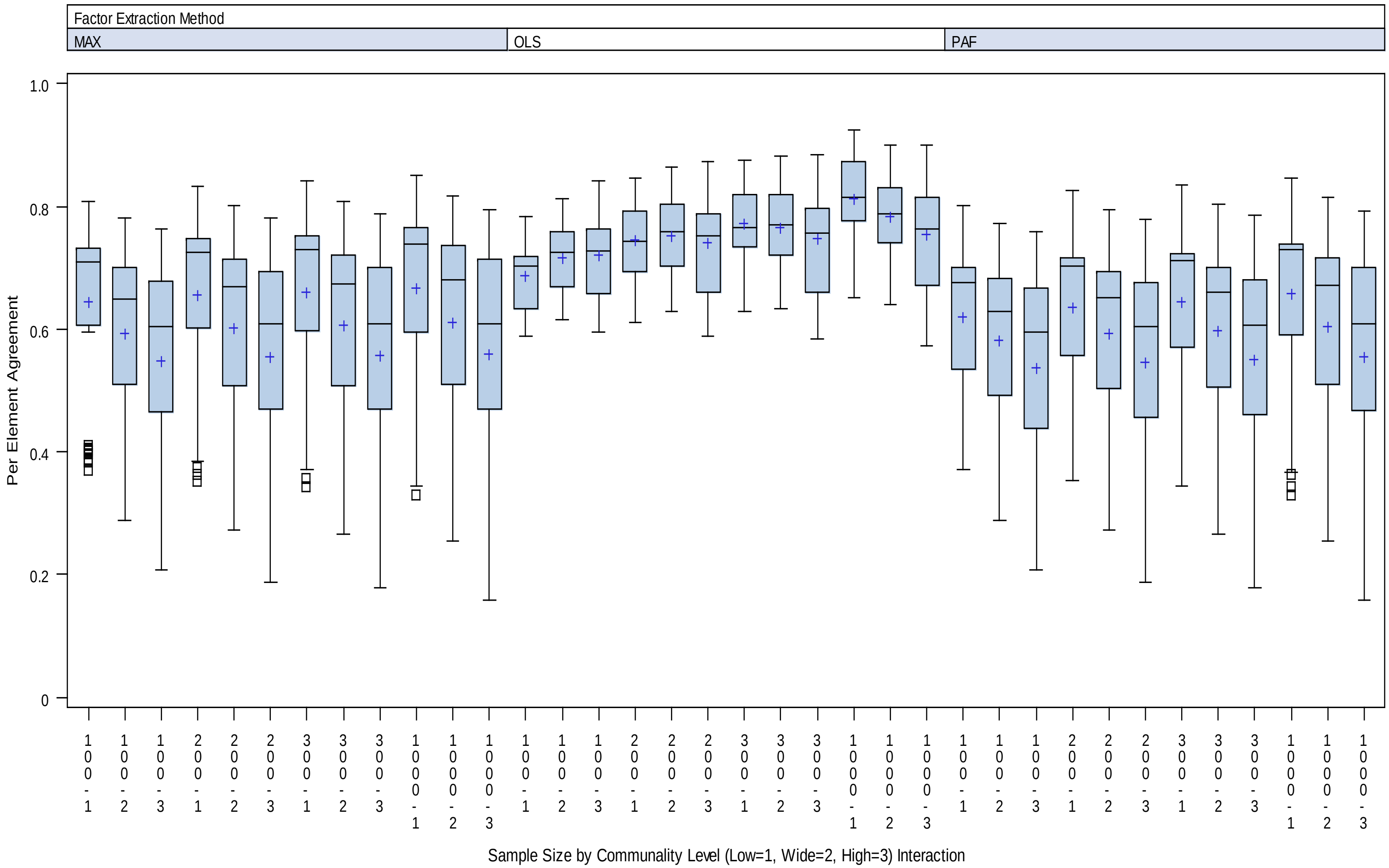


*Figure 9.* Per element factor pattern agreement by the interaction between sample size and level of communality

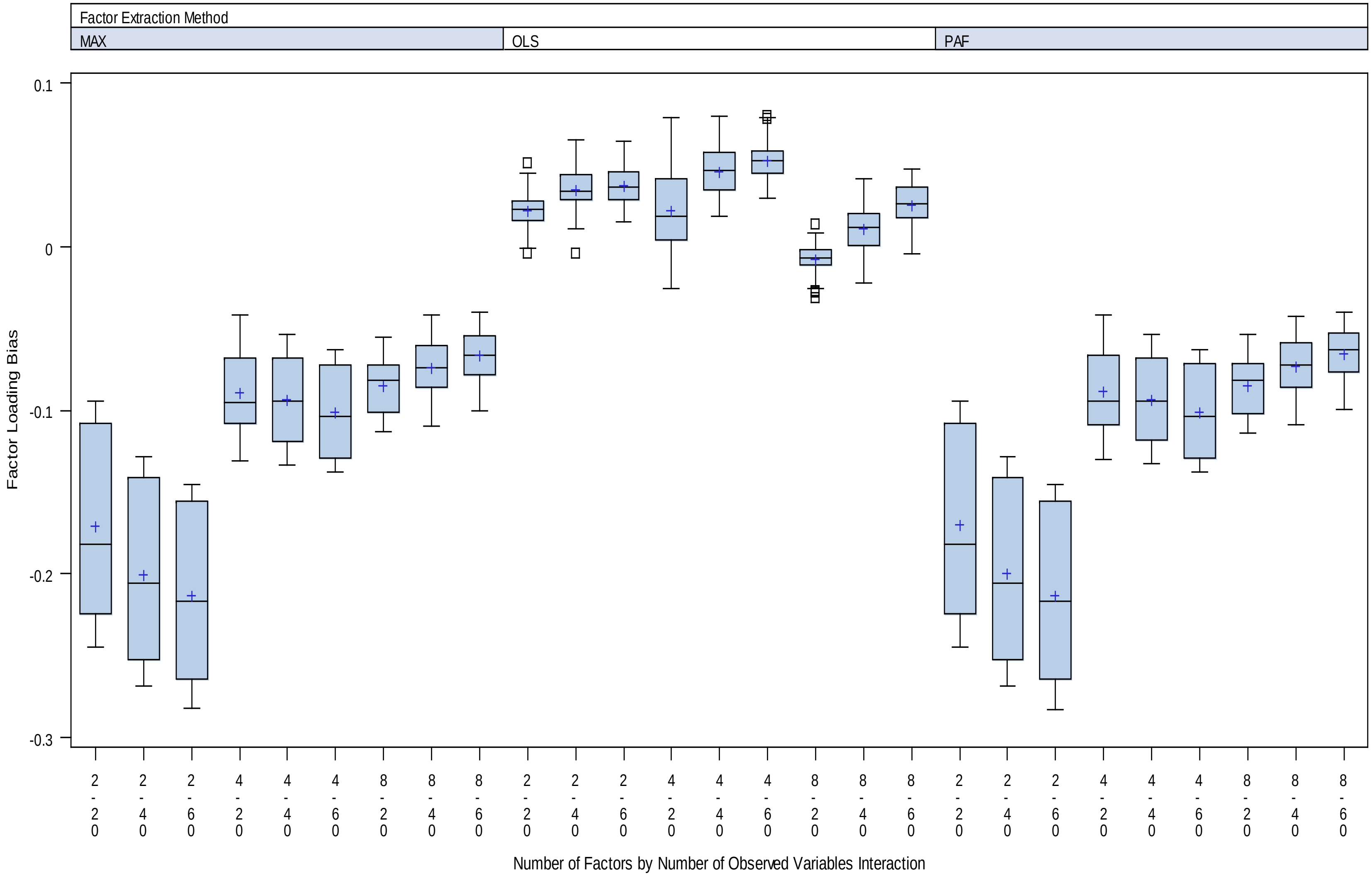


*Figure 10.* Factor loading bias by the interaction between the number of factors and number of observed variables

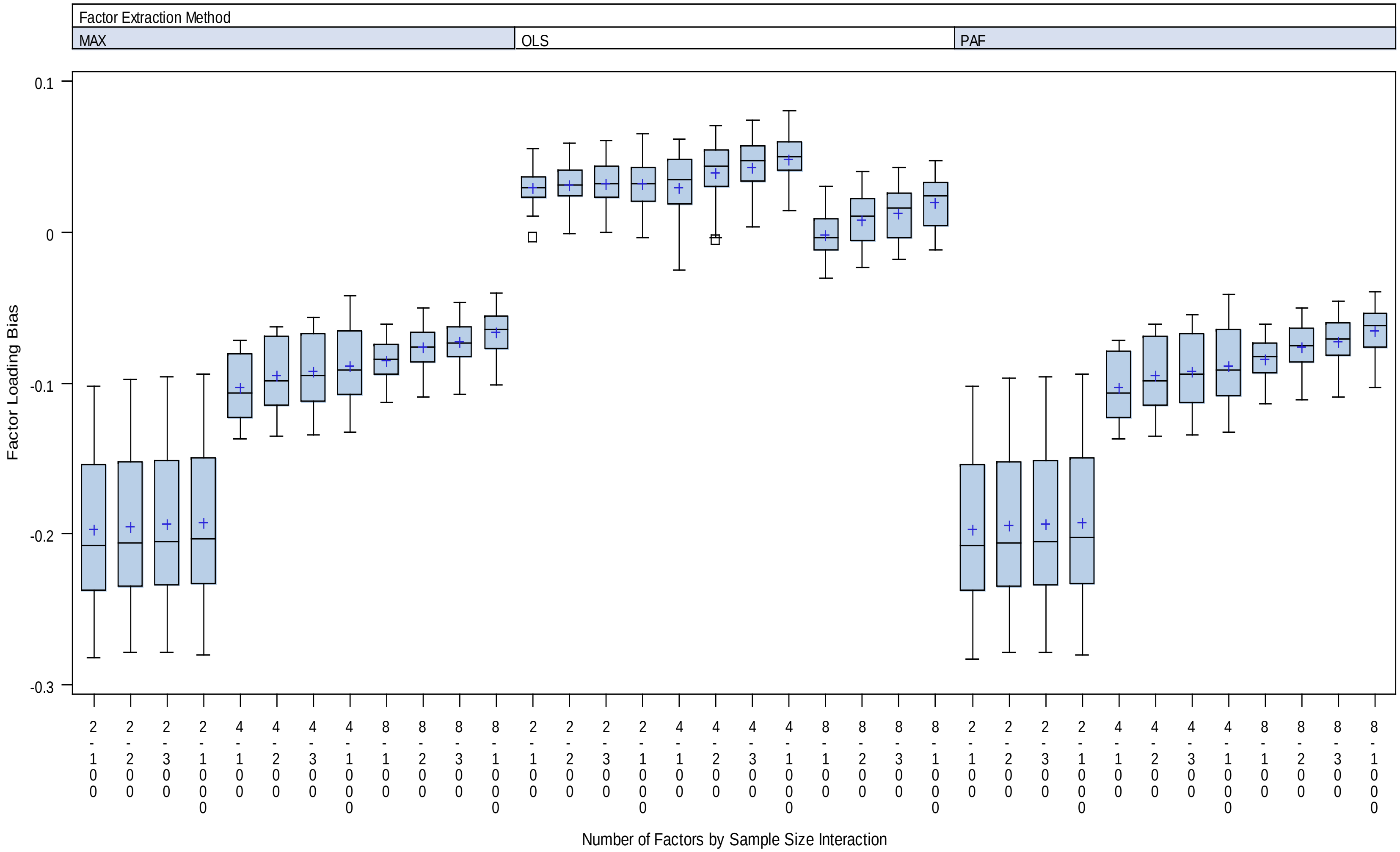


*Figure 11*. Factor loading bias by the interaction between the number of factors and sample size

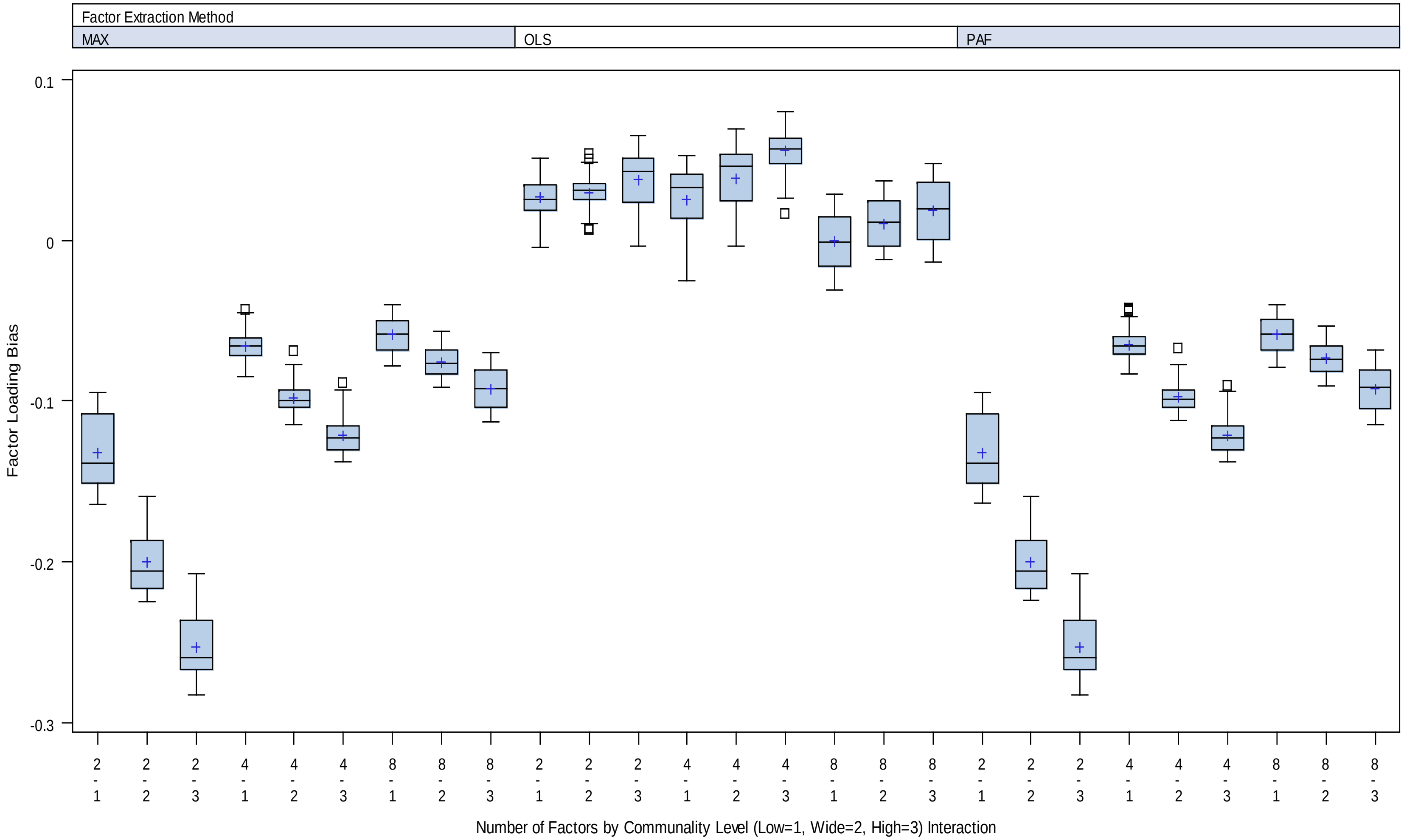


*Figure 12.* Factor loading bias by the interaction between the number of factors and communality

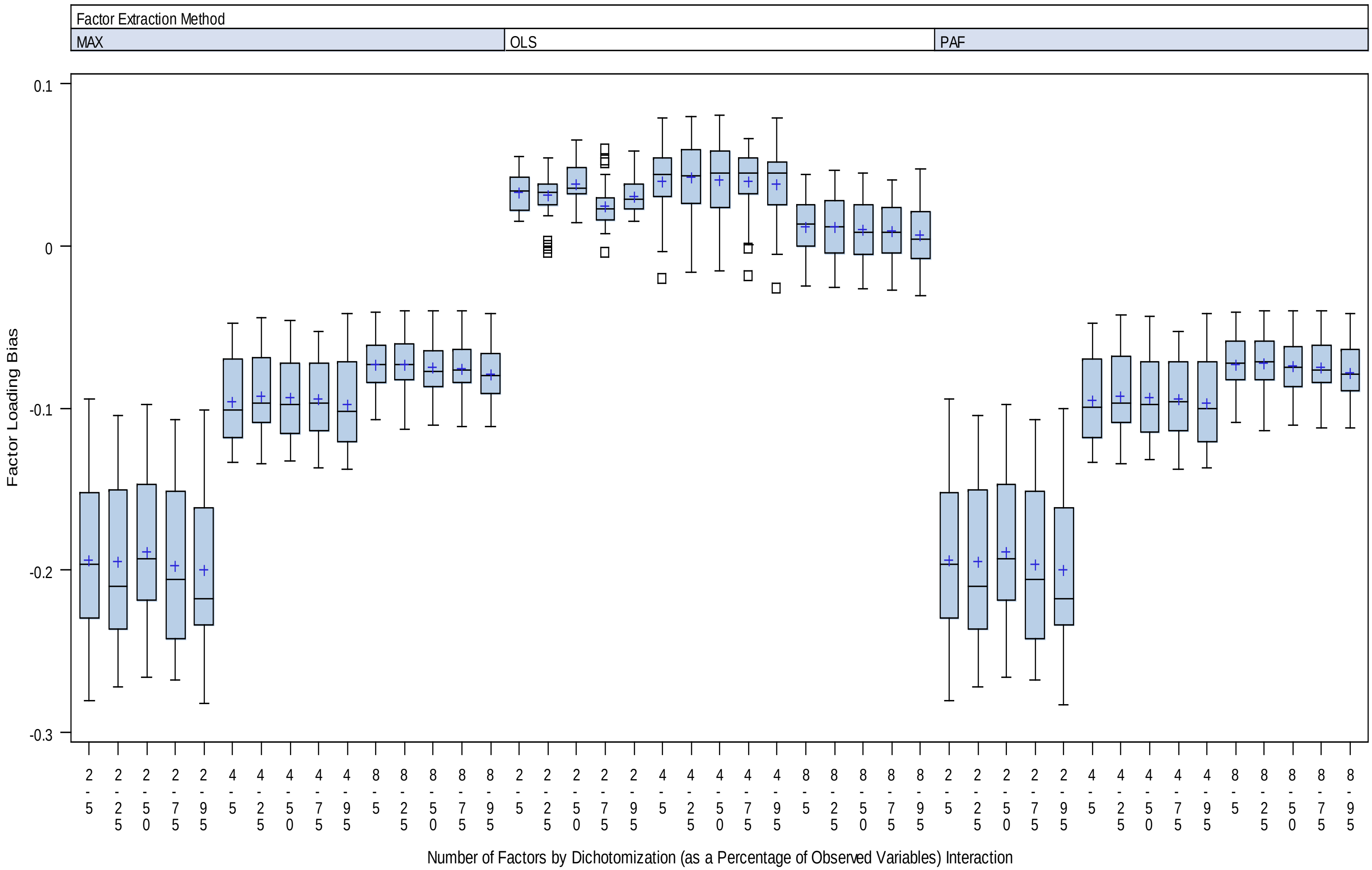


*Figure 13.* Factor loading bias by the interaction between the number of factors and level of dichotomization